



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

TX 511 P.10038
Milne, William James,
Elementary arithmetic on the inductive p

Stanford University Libraries



3 6105 04927 2573

DEPARTMENT OF EDUCATION
LELAND STANFORD JUNIOR UNIVERSITY

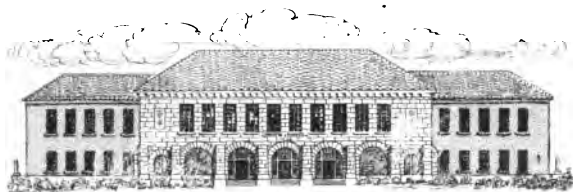
MILNE'S
ELEMENTARY ARITHMETIC



A. L. Bancroft & Co.,

San Francisco, Cal.

PRESENTED BY THE PUBLISHERS
— TO THE —
TEXT-BOOK COLLECTION



SCHOOL OF EDUCATION
LIBRARY

TEXTBOOK COLLECTION
GIFT OF
THE PUBLISHERS



STANFORD UNIVERSITY
LIBRARIES

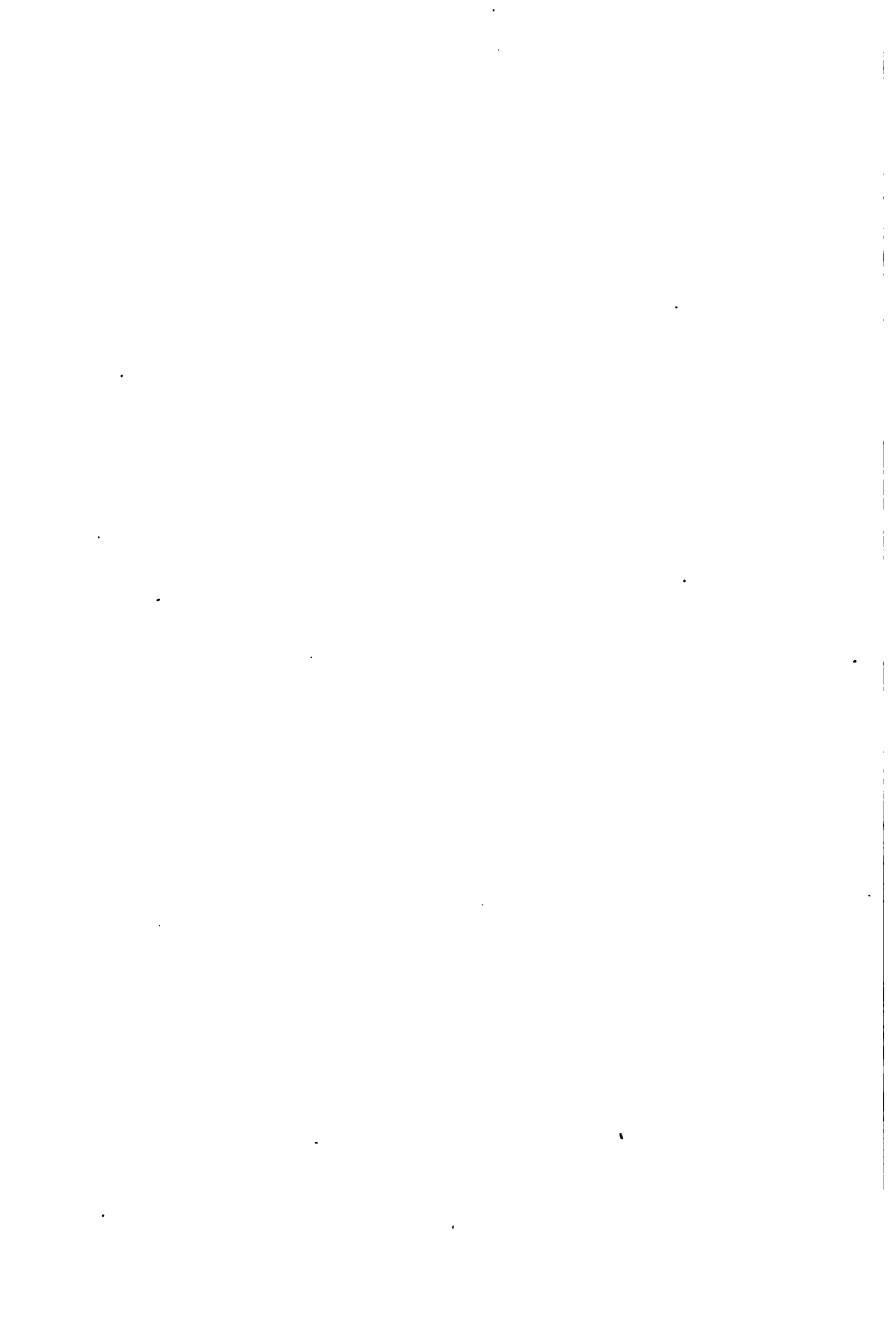
W

Louie Ockerman

Frank M. Wilson Esq.

John J. Roach Esq.

Palo Alto Stock Ranch



Inductive Series.

AN

**ELEMENTARY
ARITHMETIC**

ON THE

INDUCTIVE PLAN,

INCLUDING

ORAL AND WRITTEN EXERCISES.

BY

WILLIAM J. MILNE, Ph. D., LL. D.,
PRINCIPAL OF THE STATE NORMAL SCHOOL, GENESEO, N. Y.

**DEPARTMENT OF EDUCATION
LELAND STANFORD JUNIOR UNIVERSITY**

A. L. BANCROFT & CO.

SAN FRANCISCO, CAL.

ECLECTIC EDUCATIONAL SERIES.

615055

McGuffey's Revised Readers, Speller and Charts.

Milne's Inductive Arithmetics and Algebra.

Eclectic School Geometry.

Harvey's Revised Grammars.

Holbrook's Normal Grammars.

Ridpath's Inductive Grammar.

New Eclectic Geographies.

New Eclectic Penmanship.

Eclectic System of Drawing.

Forbriger's Drawing Tablets.

Eclectic Histories of the United States.

Thalheimer's Historical Series.

Ridpath's Histories of the United States.

Eclectic Complete Book-Keeping.

Murdoch's Analytic Elocution.

Kidd's New Elocution.

Etc., Etc., Etc.

Descriptive Catalogue and Price List on application.

COPYRIGHT, 1882, BY JOHN T. JONES.

ECLECTIC PRESS,
VAN ANTWERP, BRAGG & Co., CINCINNATI, O.



PREFACE

THIS work has been prepared at the urgent solicitation of many educators of large experience, who desire a text-book more elementary in its scope and more simple in its discussions than the author's PRACTICAL ARITHMETIC. It is designed to fill a place intermediate between the FIRST LESSONS and the PRACTICAL ARITHMETIC, yet to be complete enough to furnish all the instruction in this branch of mathematics that is sought by many pupils who attend the public schools.

The plan of the book is the same as that which has been so generally commended in the volumes of the series already published. The subjects are presented in such a way that the student is led to the apprehension of the principles of the science by natural, easy, and progressive steps, while at the same time the development and discipline of the reasoning faculties are secured rapidly and effectively. By faithfully pursuing the methods exemplified, the definitions and principles will be established *inductively*, and the spirit of investigation will be aroused, thus giving a stable foundation for further progress in any department of learning.

The number of oral examples is so large that there is no need of a "mental" arithmetic; and the reviews of principles, processes, and definitions previously discussed are so frequent and so exhaustive that the pupil at no time loses the knowledge which he has previously acquired.

All matter that could be omitted with proper regard to the rapid and intelligent progress of the student has been eliminated, and the processes have been simplified wherever it could be done without sacrificing correctness and clearness.

It is confidently believed that this work will prove to be admirably adapted for use in intermediate grades of our public schools; and that it will also supply many students in the common schools with a course extended enough to train them to transact the ordinary business of life with accuracy and intelligence, even though they pursue the science no further.

W. J. M.

STATE NORMAL SCHOOL,
GENESEO, N. Y., 1882.



CONTENTS

	PAGE
NOTATION AND NUMERATION	7
Arabic System	8
U. S. Money	17
Roman System	18
Questions for Review	20
ADDITION	21
Drill Table	35
Questions for Review	36
SUBTRACTION	37
Review Exercises	47
Questions for Review	50
MULTIPLICATION	51
Review Exercises	62
Drill Table	66
Questions for Review	67
DIVISION	68
Review Exercises	81
Questions for Review	85
FACTORING	86
CANCELLATION	89
Questions for Review	91
FRACTIONS	93
Reduction of Fractions	95
Questions for Review	101
Addition of Fractions	102
Subtraction of Fractions	105
Multiplication of Fractions	108
Division of Fractions	116

	PAGE.
Review Exercises	124
Questions for Review	130
DECIMAL FRACTIONS	131
Reduction of Decimals	134
Addition of Decimals	137
Subtraction of Decimals	138
Multiplication of Decimals	139
Division of Decimals	141
Questions for Review	144
UNITED STATES MONEY	145
ACCOUNTS AND BILLS	151
Questions for Review	154
DENOMINATE NUMBERS	155
Questions for Review	164
Reduction of Compound Numbers	165
Addition of Compound Numbers	168
Subtraction of Compound Numbers	169
Multiplication of Compound Numbers	171
Division of Compound Numbers	172
Measurements	172
Review Exercises	177
Questions for Review	181
PERCENTAGE	182
INTEREST	188
Questions for Review	191
GENERAL REVIEW	191
ANSWERS	195

ELEMENTARY ARITHMETIC

NOTATION & NUMERATION

ARTICLE 1. A **Unit** is a single thing.

Thus, one, a house, a chair, are units.

2. A **Number** is a unit or collection of units.

Thus, one, four, seven, are numbers.

3. **Arithmetic** is the science which treats of numbers and their use.

4. **Figures** are characters which are used to express numbers.

Thus, 2 expressing *two* and 3 expressing *three*, are figures.

Letters are also used to express numbers.

Thus, X represents *ten*; V, *five*; C, *one hundred*.

5. **Notation** is the method of expressing numbers by figures and letters.

6. **Numeration** is the method of reading numbers expressed by figures or letters.

ARABIC SYSTEM.

7. Arabic Notation is the method of expressing numbers by means of figures.

Its name is derived from the Arabs, by whom it was introduced into Europe.

8. In this system ten figures are employed to represent numbers, viz :

FIGURES: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

NAMES: Naught, One, Two, Three, Four, Five, Six, Seven, Eight, Nine.

Each of these, except naught, is called a *significant figure*.

Naught is also called *zero* and *cipher*.

9. The number that is *one* more than *nine* is called *ten*, or 1 ten.

1 ten is called *ten*, and is written, 10.

2 tens are called *twenty*, and are written, 20.

3 tens are called *thirty*, and are written, 30.

4 tens are called *forty*, and are written, 40.

5 tens are called *fifty*, and are written, 50.

6 tens are called *sixty*, and are written, 60.

7 tens are called *seventy*, and are written, 70.

8 tens are called *eighty*, and are written, 80.

9 tens are called *ninety*, and are written, 90.

The suffix *ty* means *ten*. Thus, *forty* means *four tens*, etc.

It will be observed that *two* figures are required to express *tens*, the one at the left expressing the number of tens.

10. By combining the ten figures according to certain principles, any number can be expressed.

11. PRINCIPLE. — *When two figures are written side by side, the one at the right expresses units, and the one at the left tens.*

EXERCISES.

12. 1. In 54, what does the 5 express? What does the 4 express? Read the number, beginning at the left.

2. In 34, what does the 3 express? What does the 4 express? Read the number, beginning at the left.

3. In 42, what does the 4 express? What does the 2 express? Read the number, beginning at the left.

4. Tell what each figure in the following expresses, and read each number, beginning at the left:

46	57	54	32	24	48
37	62	39	87	56	35
29	83	28	65	78	65
81	44	76	92	89	77
30	20	60	90	50	80

13. Numbers between 1 ten and 2 tens are named thus:

- 1 ten and 1 unit, or 11, eleven.
- 1 ten and 2 units, or 12, twelve.
- 1 ten and 3 units, or 13, thirteen.
- 1 ten and 4 units, or 14, fourteen.
- 1 ten and 5 units, or 15, fifteen.
- 1 ten and 6 units, or 16, sixteen.
- 1 ten and 7 units, or 17, seventeen.
- 1 ten and 8 units, or 18, eighteen.
- 1 ten and 9 units, or 19, nineteen.

The words thirteen, fourteen, fifteen, etc., mean *three* and *ten*, *four* and *ten*, *five* and *ten*, etc.

14. In reading the numbers the word *and*, between the tens and the units, and the word *units* are omitted.

Thus, 43 is read forty-three instead of forty *and* three *units*.

EXERCISES.

15. Read the following:

31	27	30	82	88	43
24	83	44	70	33	98
16	46	25	34	60	90
29	25	47	26	84	37
36	18	21	96	72	20

Express in figures the following:

Two tens and five units.	Sixty-four.	Thirty-two.
Five tens and six units.	Fifty-seven.	Eighty-one.
Three tens and eight units.	Thirty-five.	Fifty-eight.
Six tens and seven units.	Forty-three.	Twenty-seven.
Nine tens and two units.	Ninety-six.	Forty-eight.
Seven tens and eight units.	Seventy-nine.	Sixty-seven.
Four tens.	Eighty.	Sixty.

16. *Ten tens are called one hundred.*

1 hundred is written 100.	5 hundreds are written 500.
2 hundreds are written 200.	6 hundreds are written 600.
3 hundreds are written 300.	7 hundreds are written 700.
4 hundreds are written 400.	8 hundreds are written 800.
9 hundreds are written 900.	

17. PRINCIPLE. — *When three figures are written side by side, the one at the right expresses units, the next tens, and the next hundreds.*

18. In reading numbers expressed by *three* figures, the tens are read after the hundreds without the word *and*.

Thus, 347 is read three hundred forty-seven instead of three hundred *and* forty-seven.

19. Read the following :

316	243	269	984	856
247	819	384	399	346
385	207	569	418	200
409	539	486	209	810
235	400	397	398	401
618	619	429	426	314
842	820	899	489	264
715	457	495	924	408

Express in figures the following :

Three hundred twenty-five. Eight hundred twenty-one.

Seven hundred sixty-seven. Six hundred four.

Four hundred eighty-two. Nine hundred twenty.

Six hundred eighteen. Seven hundred eighty-nine.

Three tens, five units.

Three hundreds, eight tens, five units.

Five hundreds, seven tens, nine units.

Six hundreds, four units.

Eight hundreds, eight tens.

Seven hundreds, six tens, eight units.

Eight hundreds, five tens, nine units.

20. Figures in *units'* place express *units of the first order* ; those in *tens'* place, *units of the second order* ; those in *hundreds'* place, *units of the third order*.

EXERCISES.

21. Express in figures the following, and read the numbers :

Three units of the first order, two of the second, and three of the first.

Five units of the third order, four of the second, and two of the first.

Four units of the third order, two of the second, and one of the first.

Eight units of the third order, seven of the second, and two of the first.

Nine units of the third order, nine of the second, and nine of the first.

Eight units of the third order and one of the first.

Five units of the third order and three of the first.

Five units of the third order and two of the second.

Eight units of the third order and five of the second.

Nine units of the third order.

Six units of the third order.

From the previous exercises the following general principles of notation may be deduced :

22. PRINCIPLES.—1. *The representative value of a figure is increased tenfold by each removal one place to the left, and decreased tenfold by each removal one place to the right.*

2. *The figure 0 is used to give significant figures their positions.*

23. In reading numbers a *new name* is given to the order *next higher than hundreds* of any denomination.

Thus, the order next higher than hundreds is called *thousands*; that next higher than hundreds of thousands, *millions*; that next higher than hundreds of millions, *billions*, etc. Therefore,

24. Each denomination can have but *three orders* of units.

25. A **Period** is a group of figures containing the *hundreds, tens, and units* of any denomination.

26. Since *ten* units make one ten, *ten* tens make one hundred, and *ten* hundreds make one thousand, etc., it is evident that *ten units of any order make one of the next order*. Hence, the Arabic system is called the *decimal* system of notation.

Decimal is from the Latin word *decem*, which means *ten*.

27. The present system of notation is illustrated by the following

TABLE.

PERIODS.	5th.	4th.	3d.	2d.	1st.
NAMES.	Trillions.		Billions.		Millions.
ORDERS.	Tens Units		Hundreds Tens Units		Thousands. Tens Units
NUMBER.	4 6, 1		4 0, 3		0 8, 2 9 4, 0 4 0.

This number is read *forty-six trillion, one hundred forty billion, three hundred eight million, two hundred ninety-four thousand, forty.*

1. In reading numbers the name of units' period is omitted.
2. Each period except the highest must contain three figures.
3. The periods are separated from each other by commas.

EXERCISES.

28. Give the number of each of the following periods:

Thousands.	Trillions.	Millions.
Units.	Billions.	Units.
Millions.	Thousands.	Trillions.

Give the names of the following:

5th period.	1st period.	4th period.
3d period.	3d period.	2d period.
4th period.	5th period.	5th period.
2d period.	2d period.	3d period.

Repeat the names of the periods in order from units to billions. From units to trillions. From thousands to trillions. From millions to trillions. From thousands to billions. From trillions to units. From billions to thousands.

29. Copy and point off into periods:

- | | | |
|--------------|--------------|--------------------|
| 1. 46825. | 5. 38420058. | 9. 5284325684. |
| 2. 239746. | 6. 33468204. | 10. 7932468512. |
| 3. 180040. | 7. 8438206. | 11. 83749275867. |
| 4. 14168843. | 8. 436784. | 12. 1423789276586. |

13. How many thousands are there in the first number?

14. How many thousands in the second number?

15. How many billions in the next to the last number?

16. How many trillions in the last number? How many billions? How many millions? How many thousands? How many units?

17. Point off into periods, and name in order, the billions, millions, thousands, and units of the next to the last number?

18. Point off into periods, and name in their order, the periods composing the 12th number.

19. In like manner point off and read each of the numbers.

30. RULE FOR NUMERATION.—*Begin at the right and separate the number into periods of three figures each.*

Begin at the left and read each period as if it stood alone, adding its name.

EXERCISES.

31. Copy, point off, and read:

- | | | |
|----------|----------|-----------|
| 1. 6845. | 5. 8046. | 9. 8146. |
| 2. 7045. | 6. 7104. | 10. 8004. |
| 3. 4869. | 7. 4856. | 11. 8140. |
| 4. 8413. | 8. 7329. | 12. 7302. |

13. 5684.	22. 36000.	31. 2040605.
14. 23817.	23. 30404.	32. 2008005.
15. 34620.	24. 81643.	33. 32045068.
16. 30062.	25. 30404.	34. 41204004.
17. 45106.	26. 80464.	35. 36130006.
18. 73040.	27. 713463.	36. 30000040.
19. 46036.	28. 704306.	37. 51600406.
20. 40040.	29. 345043.	38. 568304680.
21. 31004.	30. 2345604.	39. 600480030.

32. 1. Write in figures twenty-five million, eighteen thousand, forty.

PROCESS.

3d. 2d. 1st.
 $\left| \begin{array}{c|c|c} 25 & 018 & 040 \end{array} \right|$

Or,

25,018,040.

ANALYSIS.—Since the highest denomination is millions, which occupy the third period, we make three spaces for periods. 25 is then written in the third period, thus expressing the millions of the number; 18 in the second period, thus expressing the thousands; and 40 in the first period, thus expressing the units.

Since every period except the highest must contain three figures, the vacant places are filled with ciphers.

As soon as possible use commas instead of lines, and cease to write the number and name of the periods.

RULE FOR NOTATION.—*Begin at the left, and write the hundreds, tens, and units of each period in their proper order, putting ciphers in all vacant places and periods.*

While writing, separate each period from the next by a comma.

Write in figures:

2. Six thousand, eight hundred forty.
3. Eight thousand, two hundred fifty-seven.
4. Nineteen thousand, five hundred twenty-seven.
5. Eighty-four thousand, six hundred thirty-one.
6. Fifty-eight thousand, seven hundred twenty-eight.
7. Two hundred thousand, four hundred seventy-five.

8. Four hundred eighteen thousand, two hundred forty-eight.

9. Five hundred twenty-seven thousand, eight hundred four.

10. Seven hundred fifteen thousand, nine hundred, twenty-three.

11. Thirty-two million, four hundred fifteen thousand, two hundred twenty one.

12. Forty-six million, eight hundred forty-nine thousand, six hundred forty.

13. Two hundred sixty-five million, fifteen thousand, one hundred twenty-six.

14. Three hundred sixty-five million, forty-nine.

15. Thirty-six million, eighteen thousand.

16. Twenty-four million, eight thousand, eight.

17. Two hundred million, two thousand, two.

18. Twenty-nine billion, one thousand, eight hundred.

19. Fifty-nine billion, one million, one thousand, one.

20. Six hundred twenty million, eighty-four thousand.

21. Four hundred sixty-seven million, nine thousand, nine.

22. Fifty billion, fifty million, fifty thousand, fifty.

23. Six hundred billion, six hundred million, six hundred.

24. Eighty-six billion, nineteen million, twenty-four thousand.

25. Ninety-seven billion, eighty-four thousand, five.

26. Four hundred twenty-six million, ten.

27. Three hundred twenty-five billion, eighteen million, ninety thousand, nine hundred ninety.

28. Five hundred twenty-six million, four hundred thousand, twenty-five.

29. Six hundred twenty-five million, eighteen thousand, eight hundred eighty-eight.

30. Four hundred twenty-eight thousand, nine hundred four.

31. Four hundred million, three hundred twenty-nine.
32. Fifty-five billion, two hundred million, nine hundred eighty-four thousand, seventy-five.
33. Six hundred ninety-five million, eight hundred thousand, four hundred forty-one.
34. Seventy million, seventy thousand, seven.
35. Eighty-five billion, eight million, seven thousand two hundred ninety-four.
36. Fifty-two billion, two hundred six million, forty thousand, eight hundred five.

NOTATION AND NUMERATION OF UNITED STATES MONEY.

33. In the currency of the United States

- 10 mills make 1 cent.
- 10 cents make 1 dime.
- 10 dimes make 1 dollar.
- 100 cents make 1 dollar.

34. The **Sign of Dollars** is \$. It is written before the number.

Thus, \$24 is read, twenty-four dollars.

35. In writing cents and mills a *period* called the *decimal point* is placed before the number.

Cents occupy the *first two* places at the right of the decimal point, and *mills* the *third*.

Thus, \$.24 is read, twenty-four cents; \$3.185 is read, three dollars, eighteen cents, five mills.

36. If the number of cents is less than ten, a cipher must be written in the first place at the right of the decimal point.

Thus, four dollars, three cents, five mills, is written, \$4.035.

EXERCISES.

37. Read the following:

\$24.16.	\$.46.	\$318.273.
\$35.28.	\$.375.	\$214.384.
\$2.357.	\$.286.	\$315.273.
\$1.065.	\$2.83.	\$356.246.
\$15.43.	\$2.45.	\$218.392.
\$13.29.	\$5.645.	\$184.169.

Write the following:

1. Eighteen dollars, twenty-four cents.
2. Twenty-six dollars, thirty-six cents.
3. Fifty-one dollars, five cents, five mills.
4. Thirty-eight dollars, six cents, three mills.
5. Twenty-seven cents, eight mills.
6. Thirty-four cents, nine mills.
7. Eighty-six dollars, five cents, two mills.
8. Twenty-one dollars, twenty-one cents, one mill.
9. Thirty-seven dollars, eighteen cents, eight mills.
10. Forty-nine dollars, nine cents, six mills.

ROMAN SYSTEM.

38. **Roman Notation** is the method of expressing numbers by means of letters.

It is called the Roman Notation because it was originally used by the ancient Romans.

39. In this system seven capital letters are used to express numbers, viz:

LETTERS: I, V, X, L, C, D, M.

VALUES: 1, 5, 10, 50, 100, 500, 1000.

40. By combining these letters according to certain principles, any number can be expressed.

PRINCIPLES.—1. *When a letter is repeated its value is repeated.*

Thus, X represents ten; XX, twenty; XXX, thirty; C, one hundred; CC, two hundred.

2. *When a letter is placed before another of greater value its value is to be taken from that of the greater.*

Thus, I represents one and V five, but IV represents four; IX, nine; XIX, nineteen; XL, forty; XC, ninety.

3. *When a letter is placed after another of greater value their values are to be united.*

Thus, XV represents fifteen; LXX, seventy; LXXX, eighty; DC, six hundred.

4. *A bar placed over a letter increases its value a thousand-fold.*

Thus, V represents five; \overline{V} , five thousand; LXX represents seventy; \overline{LXX} , seventy thousand.

TABLE.

I	1	XIV	14	LX	60
II	2	XV	15	LXX	70
III	3	XVI	16	LXXX	80
IV	4	XVII	17	XC	90
V	5	XVIII	18	C	100
VI	6	XIX	19	CC	200
VII	7	XX	20	CCL	250
VIII	8	XXI	21	CCCC	400
IX	9	XXIX	29	D	500
X	10	XXX	30	DCC	700
XI	11	XXXIV	34	M	1000
XII	12	XL	40	MMM	3000
XIII	13	L	50	MDCCLXXX	1880

41. Read the following:

XX; XXI; XXXV; XXV; XL; LX; LXX; XIX; LXIX;
LXIV; XC; XCIV; XCIX; XLIV; CDXX; CCXXIV; VIII;
XIX; MDLIV.

Express the following by Roman Notation:

23, 34, 15, 27, 61, 36, 43, 84, 59, 62, 35, 47, 86, 214,
312, 419, 226, 384, 216, 493, 499, 276.

QUESTIONS FOR REVIEW.

42. What is a unit? What is a number? What is arithmetic? What are figures? What beside figures are used to express numbers? What is notation? What is numeration? What is Arabic notation? Why is it called so? How many figures are employed in this system? What is each figure, except naught, called? What other names has naught? What does *ty* mean in such words as *ninety*, etc.? What is the principle relating to expressing numbers by two figures? What does *teen* in the words *thirteen*, etc., mean? In reading numbers expressed by two figures, what words are omitted?

What is the principle relating to expressing numbers by three figures? How are such numbers read? Why is the Arabic system of notation called the decimal system? From what is the word *decimal* derived? What are the general principles of notation? How many orders of units are there in each denomination? What is a period? What are the names of the first five periods? What is the rule for numeration? What is the rule for notation?

In the currency of the United States how many mills make a cent? How many cents make a dime? How many dimes make a dollar? How many cents make a dollar? What is the sign of dollars? Where is it written? How are cents and mills distinguished from dollars? How are the cents expressed if the number is less than ten?

What is Roman notation? What are used in this system to express numbers? What are the principles of this system of notation?

ADDITION

INDUCTIVE EXERCISES.

43. 1. How many pens are 2 pens and 4 pens?
2. James spent 4 cents for an orange and 3 cents for apples. How much did he spend for both?
3. Mr. Davis had 3 white horses and 4 bay horses. How many horses had he?
4. Emma knit 3 pairs of gloves for her brothers and 2 pairs for her cousins. How many pairs did she knit?
5. A girl learned 5 songs at school and 3 at home. How many songs did she learn?
6. A market woman sold a boy some apples for 5 cents and some pears for 3 cents. How much did he pay her for both.
7. There were 4 birds' nests on one tree and 2 on another. How many were there on both?
8. James spelled 8 words correctly and misspelled 3 words. How many words were there in his lesson?
9. A house has 3 outside doors and 5 doors on the inside. How many doors are there in the house?
10. There are 5 fingers on each hand. How many are there on both hands?
11. Jane read 3 pages of a book in the forenoon and 4 in the afternoon. How many pages did she read?
12. Samuel had 4 marbles and bought 4 more. How many had he then?

13. A lad saw 4 squirrels on one tree and 6 on another. How many squirrels did he see?

14. The distance to my uncle's house is 4 miles and it is 7 miles farther to my grandfather's. How far is it to my grandfather's?

15. How many are 5 oranges and 3 oranges? 6 boys and 3 horses?

16. Why can you tell how many 5 oranges and 3 oranges are, while you can not tell how many 6 boys and 3 horses are?

17. What kind of numbers can be united?

Numbers that express things, of the same kind are called **Like Numbers**.

DEFINITIONS.

44. **Addition** is the process of finding a number that is equal to two or more given numbers.

45. The **Sum** or **Amount** is the result obtained by adding.

46. The **Sign of Addition** is a small upright cross: $+$. It is called *plus*, and is placed between the numbers to be added.

Thus, $4 + 2$ is read 4 plus 2, and means that 4 and 2 are to be added.

47. The **Sign of Equality** is two short parallel horizontal lines: $=$. It is read *equals*, or *is equal to*.

Thus, $3 + 2 = 5$ is read 3 plus 2 equals 5.

The expression $3 + 2 = 5$, or any other expression of equality, is called an **Equation**.

48. **PRINCIPLES**.—1. *Only like numbers can be added.*

2. *The sum and numbers added must be like numbers.*

TABLE.

1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6
2 + 1 = 3	2 + 2 = 4	2 + 3 = 5	2 + 4 = 6	2 + 5 = 7
3 + 1 = 4	3 + 2 = 5	3 + 3 = 6	3 + 4 = 7	3 + 5 = 8
4 + 1 = 5	4 + 2 = 6	4 + 3 = 7	4 + 4 = 8	4 + 5 = 9
5 + 1 = 6	5 + 2 = 7	5 + 3 = 8	5 + 4 = 9	5 + 5 = 10
6 + 1 = 7	6 + 2 = 8	6 + 3 = 9	6 + 4 = 10	6 + 5 = 11
7 + 1 = 8	7 + 2 = 9	7 + 3 = 10	7 + 4 = 11	7 + 5 = 12
8 + 1 = 9	8 + 2 = 10	8 + 3 = 11	8 + 4 = 12	8 + 5 = 13
9 + 1 = 10	9 + 2 = 11	9 + 3 = 12	9 + 4 = 13	9 + 5 = 14
1 + 6 = 7	1 + 7 = 8	1 + 8 = 9	1 + 9 = 10	1 + 10 = 11
2 + 6 = 8	2 + 7 = 9	2 + 8 = 10	2 + 9 = 11	2 + 10 = 12
3 + 6 = 9	3 + 7 = 10	3 + 8 = 11	3 + 9 = 12	3 + 10 = 13
4 + 6 = 10	4 + 7 = 11	4 + 8 = 12	4 + 9 = 13	4 + 10 = 14
5 + 6 = 11	5 + 7 = 12	5 + 8 = 13	5 + 9 = 14	5 + 10 = 15
6 + 6 = 12	6 + 7 = 13	6 + 8 = 14	6 + 9 = 15	6 + 10 = 16
7 + 6 = 13	7 + 7 = 14	7 + 8 = 15	7 + 9 = 16	7 + 10 = 17
8 + 6 = 14	8 + 7 = 15	8 + 8 = 16	8 + 9 = 17	8 + 10 = 18
9 + 6 = 15	9 + 7 = 16	9 + 8 = 17	9 + 9 = 18	9 + 10 = 19

SIGHT ADDITION.

3	6	5	4	3	5	4	5	6	5	8	3
5	4	6	8	9	3	2	7	4	6	5	8
4	5	8	9	8	4	5	4	5	6	4	8
7	3	4	5	6	8	3	9	7	8	7	5
5	4	3	8	6	7	4	8	9	8	9	9
7	6	9	5	9	5	6	5	7	8	6	9

Additional exercises in sight addition may be had from Drill Table, page 35.

ORAL EXERCISES.

49. 1. A man spent \$6 for shoes and \$3 for a hat. How much did he pay for both?

2. A boy paid 5 cents for a pencil and 4 cents for some paper. How much did he pay for both?

3. Mary solved 5 examples and Carrie solved 6. How many did both solve?

4. I paid \$9 for a barrel of flour and \$6 for a ton of coal. How much did I pay for both?

5. A lad earned 9 cents on Monday and 6 cents on Tuesday. How much did he earn on both days?

6. Harry is 5 years old, and his sister is 6 years older than he. How old is his sister? ♡

7. In one field there are 5 sheep and in another there are 7. How many are there in both?

8. On one branch of a rose-bush there were 7 roses and on another there were 8. How many roses were there on both branches?

9. A laborer dug 8 rods of ditch the first day and 9 rods the second day. How many rods did he dig in both days?

10. A boy spent 5 cents for candy and 7 cents for nuts. How much did he spend?

11. A horse traveled 9 miles the first hour and 8 miles the second hour. How far did he go in the two hours? ♡


12. Jane purchased a yard of muslin for 9 cents and a spool of thread for 5 cents. How much did she pay for both?

13. A man paid \$2 for paper, \$3 for books, and \$4 for a hat. How much did he spend?

14. A farmer who had 6 horses, bought at one time 5 more and at another 3 more. How many horses did he then have?

15. A boy learned 5 verses of a piece of poetry in the

morning, and the rest, which was 7 verses, in the afternoon. How many verses were there?

16. A house has 5 windows on one side, 7 on another, and 3 in each end. How many windows are there in the house? 

17. A fruit dealer sold James 7 oranges, Henry 5 oranges, and John 4 oranges. How many oranges did he sell them?

18. Horace caught 5 fishes, Henry 6, and William 7. How many did they all catch?

19. Add by 2's from 0 to 20. Thus, 0, 2, 4, 6, 8, 10, etc.

20. Add by 2's from 1 to 27. From 3 to 35.

21. Add by 3's from 0 to 30. From 4 to 37.

22. Add by 3's from 1 to 37. From 6 to 42.

23. Add by 5's from 0 to 40. From 1 to 51.

24. Add by 5's from 2 to 47. From 3 to 63.

25. What is the sum of 2, 3, and 5? Of 2, 5, and 4?

26. What is the sum of 4, 2, and 6? Of 4, 3, and 6?

27. What is the sum of 2, 5, and 7? Of 2, 8, and 7?

28. Mary had a ten-cent piece, a five-cent piece, a three-cent piece, and 4 one-cent pieces. How much money had she?

29. Henry's hens laid 4 eggs on Monday, 3 on Tuesday, and 4 on Wednesday. How many eggs did they lay in the three days?

30. A cat killed 4 mice one week, 5 the next, and 6 the next. How many mice did she kill in the three weeks?

31. A boy rode on his bicycle 5 miles the first hour, 7 miles the second, and 6 miles the third. How far did he ride in those three hours?

32. On one chandelier there are 4 burners, on another 6, and on another 3. How many burners are there on the three chandeliers?

33. A little girl paid 5 cents for some ribbon, 6 cents for lace, and 5 cents for a flower. How much did she pay for all?

(12.)	(13.)	(14.)	(15.)	(16.)	(17.)	(18.)	(19.)	(20.)	(21.)
4	5	6	2	4	2	6	5	4	3
5	8	8	7	2	4	8	9	6	9
3	4	4	9	8	5	3	8	2	2
2	2	3	6	4	8	7	6	8	5
6	7	5	5	3	4	4	5	7	4
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>

Required the sum of the following:

- | | |
|---------------------------|---------------------------|
| 22. 6, 8, 4, 2, 5, and 4. | 28. 6, 5, 4, 9, 7, and 6. |
| 23. 3, 5, 8, 4, 6, and 7. | 29. 2, 8, 6, 3, 9, and 4. |
| 24. 2, 4, 6, 5, 3, and 9. | 30. 5, 2, 8, 4, 3, and 7. |
| 25. 3, 2, 5, 8, 6, and 8. | 31. 4, 2, 6, 5, 7, and 4. |
| 26. 4, 5, 6, 8, 3, and 4. | 32. 8, 7, 9, 3, 6, and 2. |
| 27. 4, 5, 1, 3, 9, and 8. | 33. 5, 8, 4, 3, 4, and 4. |

34. James is 4 years old, Henry is 4 years older, William is 5 years older than Henry, and Samuel is 3 years older than William. How old is Samuel?

35. A tinsmith made 4 milk pans per hour for 5 hours. How many did he make in that time?

36. A pole stands 5 feet in the mud, 5 feet in the water, and 7 feet above the water. What is the length of the pole?

37. I counted the children sitting on four benches. On the first there were 6 girls, on the second 5 boys, on the third 6 boys, and on the fourth 6 girls. How many children were there on the four benches?

38. Henry had 5 cents, James had 7 cents, William had 9 cents, and Thomas had 5 cents. They paid all they had for a ball. How much did it cost them?

39. At a school entertainment 7 boys read compositions, 5 girls recited pieces of poetry, 4 other boys and 4 other girls sung. How many children took part in the exercises?

40. The children of a school having a holiday, 5 boys and 3 girls went skating, 6 girls and 4 boys went coasting,

and the rest of them, 5 boys, built snow forts. How many children were there in the school?

41. James solved 7 examples on Monday, 8 on Tuesday, and 5 on Wednesday. How many did he solve?

42. A boy had 7 copper cents, 2 silver three-cent pieces, 2 nickel three-cent pieces, 1 silver five-cent piece, and 1 nickel five-cent piece. How much money had he?

43. A boy saw four flocks of wild geese. The first contained 7 geese, the second 9, the third 8, and the fourth 9. How many wild geese did he see?

44. The distance from Alburn to Grade is 6 miles, from Grade to Newton 8 miles, from Newton to Arnold 9 miles, and from Arnold to Houston 8 miles. How far is it from Alburn to Houston?

45. A school-boy wrote 8 lines in his copy-book on Monday, 7 on Tuesday, 5 on Wednesday, 7 on Thursday, and 6 on Friday. How many lines did he write during the week?

ORAL EXERCISES.

51. 1. Count by 10's from 0 to 100; thus, 0, 10, 20, 30, 40, 50, etc.

2. Count by 10's from 5 to 95. From 7 to 57.

3. Count by 10's from 8 to 78. From 4 to 94.

4. Count by 10's from 6 to 86. From 3 to 103.

5. Count by 20's from 0 to 80. From 10 to 90.

6. Count by 20's from 2 to 62. From 4 to 84.

7. Count by 20's from 5 to 105. From 7 to 107.

8. Henry paid 5 cents for a top, 10 cents for a slate, and 10 cents for a writing-book. How much did he pay for all?

9. A carriage-maker employed 7 wood-workers, 10 iron-workers, and 10 painters. How many men did he employ?

10. A lad caught 10 trout, 8 bass, and 9 perch. How many fish did he catch?

11. Mr. Davis gave his son 15 cents, and his daughter 10 cents. How many cents did he give both?

12. I paid 8 cents for raisins, 10 cents for plums, and 20 cents for currants. How much did I pay for all?

13. A boy bought a velocipede for \$18, and a watch for \$10. How much did he pay for both?

14. Mary read 20 pages of history one day, 15 pages the next, and 10 the next. How many pages did she read in all?

15. Henry solved 22 oral examples, and 20 written examples. How many examples did he solve?

16. In a school there were 31 girls, and $(20 + 7)$ or 27 boys. How many pupils were there in the school?

17. How many are 7 and 5? 17 and 5? 27 and 5? 37 and 5? 47 and 5? 57 and 5? 67 and 5? 77 and 5?

18. How many are 8 and 6? 18 and 6? 28 and 6? 38 and 6? 48 and 6? 58 and 6? 68 and 6? 78 and 6?

19. How many are 9 and 5? 9 and 15? 9 and 25? 9 and 35? 9 and 45? 9 and 55? 9 and 65? 9 and 75?

20. How many are 21 and 8? 21 and 18? 21 and 28?

21. How many are 36 and 10? 36 and 20? 36 and 30? 36 and $(30 + 6)$ or 36? 36 and 39? 36 and 40? 36 and 46?

WRITTEN EXERCISES.

52. To add several columns.

What is the sum of \$246, \$369, and \$423?

PROCESS. ANALYSIS.—For convenience the numbers are arranged
 $\begin{array}{r} \$246 \\ 369 \\ 423 \\ \hline \$1038 \end{array}$ so that units of the same order stand in the same column.
 Beginning with the lowest order of units, each column is added separately. Thus, $3 + 9 + 6 = 18$, the sum of the units. 18 units are equal to 1 ten and 8 units. The 8 is written under the column of units; the 1 is reserved to add with the tens.

1 reserved + 2 + 6 + 4 = 13, the sum of the tens. 13 tens are

equal to 1 hundred and 3 tens. The 3 tens are written under the column of tens, and the 1 is reserved to add with the hundreds.

1 reserved + 4 + 3 + 2 = 10, the sum of the hundreds. 10 hundreds are equal to 1 thousand and 0 hundreds, which are written in thousands' and hundreds' places in the sum.

Hence the sum is \$1038.

In adding, name results only. Thus, instead of saying, 3 and 9 are 12 and 6 are 18, say 3, 12, 18.

53. RULE.—*Arrange the numbers so that units of the same order stand in the same column.*

Begin at the right, add each column separately, and write the sum, if it is less than ten, under the column added.

If the sum of any column is more than ten, write the unit figure only under that column, and add the ten or tens with the next column.

Write the entire sum of the last column.

PROOF.—*Add each column in the reverse order. If the results agree, the work is probably correct.*

Copy, add, and prove:

(2.)	(3.)	(4.)	(5.)	(6.)
324	239	518	264	289
465	346	297	319	423
<u>246</u>	<u>217</u>	<u>483</u>	<u>854</u>	<u>617</u>

(7.)	(8.)	(9.)	(10.)
\$243	\$24.18	\$124.43	\$4.85
216	26.34	239.86	13.08
319	9.21	14.74	214.63
243	8.36	16.09	7.05
163	5.16	28.35	12.18
<u>212</u>	<u>3.22</u>	<u>27.42</u>	<u>23.12</u>

(11.)	(12.)	(13.)	(14.)
<i>Horses.</i>	<i>Gallons.</i>	<i>Hundredths.</i>	<i>Rods.</i>
3168	4168	2968	4
4273	469	485	48
3192	3258	8	269
<u>4265</u>	<u>419</u>	<u>923</u>	<u>19</u>

15. Add 3684, 2973, 2869, 5841, 6853.

16. Add 4857, 2964, 382, 2879, 66.

17. Add \$23.25, \$13.21, \$26.18, \$41.15, \$18.08.

18. Add \$24.18, \$3.29, \$4.08, \$3.18, \$5.79.

19. Add \$26.055, \$18.25, \$31.055, \$21.34, \$6.835.

20. Add \$168.24, \$231.19, \$318.67, \$215.36, \$48.355.

21. Add three thousand, eight hundred forty-five; five thousand, two hundred eighteen; eight thousand, forty.

22. Add fifty-four thousand, two hundred sixty-five; nine thousand, six hundred seventy-one; eighteen thousand, six hundred four; thirty-five thousand, four hundred.

23. What is the sum of two million, three hundred four thousand, eight hundred sixty-seven; eight million, five hundred thirty-nine thousand, two hundred eighty; twenty-one million, eight thousand, seven hundred eighty-eight; four hundred twenty-nine thousand, five hundred?

24. Add twenty-one dollars, fifteen cents; thirty-seven dollars, twenty-four cents; fifteen dollars, eight cents; eighty-eight dollars, twenty-nine cents.

25. Add 24 dollars, 5 cents; 18 dollars, 18 cents; 26 dollars, 39 cents; 55 dollars, 24 cents; 72 dollars, 30 cents.

26. Add fifteen dollars, seven cents, five mills; seventeen dollars, eight cents; twenty-six dollars, ninety-seven cents, eight mills.

27. A man paid \$375 for a carriage, \$250 for a horse, and \$175 for harness. How much did he pay for all?

28. Mr. B. owns three pieces of land. The first contains 345 acres, the second 482, and the third 238. How many acres in the three pieces?

29. A man owned two store-houses. In the first were stored 789 bushels of corn, 685 bushels of wheat, and 963 bushels of oats; in the second were stored 347 bushels of wheat, 751 bushels of corn, and 563 bushels of oats. How many bushels of grain were there in each store-house, and how many in both?

30. A census-taker found that the population in five towns was as follows: In the first, 2134; in the second, 1927; in the third, 3411; in the fourth, 1095; in the fifth, 2069. What was the population of all five towns?

31. A ship, on her way to Australia, sailed the following distances: The first week 789 miles, the second week 878 miles, the third week 724 miles. How far had she sailed at the end of three weeks?

32. A farmer raised the first year 560 bushels of beans, the second year 640 bushels, the third 735 bushels, the fourth 620 bushels, the fifth 700 bushels. How many bushels did he raise in all?

33. The distance from A to B is 370 miles, from B to C 465, from C to D is 329 miles. How far is it from A to D?

34. A farmer plowed 25 acres of land in one week, 30 acres the next week, and the next week as many as in both the other weeks. How many acres did he plow in all?

35. A merchant owes one creditor \$1428, another \$3065, another \$7891, and another \$4025. How much does he owe them all?

36. A man left to his eldest son \$3000, to his eldest daughter \$3500, to his youngest son \$1500, and to his youngest daughter \$2500, and there was a surplus of \$500. How large was his estate?

37. In planting potatoes, a man put 868 in the first row, 925 in the second, 788 in the third, and 800 in the fourth row. How many potatoes did he plant in the four rows?

38. Several persons contributed towards building a church. One gave \$1500, another gave \$3500, another gave \$2000, another gave \$2500, and another \$1000. How much did the five give?

39. A owns a farm of 326 acres, B one of 484 acres, and C one as large as both the others. How many acres are there in C's farm? How many acres are there in all three farms?

40. A public library in a certain city contains 15000 books; in another city there are two libraries containing together 4500 books; in another, one containing 1485 books. How many books in these libraries?

41. How far will a man travel if he goes 850 miles the first week, 688 miles the next, 550 miles the next, and 791 miles the next?

42. In a freight train of five cars, one car weighs 20350 pounds, another 22640 pounds, another 19780 pounds, another 21560 pounds, and another 18640 pounds. How many pounds do these five cars weigh?

43. If a man pays \$4652 for one house, \$3821 for another, and \$8651 for a third, what do the three houses cost him?

44. A fisherman caught 265 fish one week, 308 the next, 127 the next, and 205 the next. How many did he catch in the four weeks?

45. A silversmith used 2798 grains of silver in one week, 1068 grains the next, and 2058 the next. How many grains did he use in those weeks?

46. Bought one house and lot for \$6825, another for \$5075, and sold them so as to gain on both \$1500. How much did I receive for them?

47. There are three piles of lumber. The first contains

90765 feet, the second 75081 feet, and the third 88708 feet.

How many feet in the three piles?

48. A certain county received \$6685 of the common school fund, another \$2513, another \$2324, and another \$3233.

How many dollars did the four counties receive?

49. The number of teachers employed in one county was 283, in another 268, in another 239, in another 281, in another 290. How many teachers were employed in the five counties?

50. A man traveled 1100 miles in one month, 985 in the next, 1284 in the next, and 1070 in the next. How far did he travel in four months?

(51.)	(52.)	(53.)	(54.)
812	325	9834	4856
437	486	5768	3293
685	324	4893	4695
928	597	9127	8697
426	846	8694	8432
832	789	5231	4546
213	346	3245	3284
424	454	4163	3849
687	392	8241	2965
821	816	5234	1694
345	291	3169	7283
428	397	9287	6546
687	928	5149	2193
529	465	3285	3297
124	869	4283	8756
386	764	3268	8438
427	468	5729	5769
814	578	4253	3285
978	663	7447	8133
811	575	6682	7748

DRILL TABLE.

	A	B	C	D	E	F	G	H	I	J
1	4	5	2	3	5	4	7	5	4	4
2	2	7	8	5	2	1	9	6	8	6
3	3	2	4	6	9	6	8	8	9	8
4	5	9	1	8	4	8	4	4	3	1
5	1	8	6	2	5	5	6	5	2	3
6	6	4	5	7	8	7	5	7	1	6
7	2	2	7	9	6	9	7	9	4	8
8	7	1	2	3	7	3	4	4	5	7
9	4	6	9	8	4	2	3	6	4	9
10	3	8	1	1	5	8	9	8	3	3
11	8	5	8	7	6	4	8	9	8	2
12	6	3	4	6	1	6	2	7	2	5

HOW TO USE THE TABLE.

54. 1. FOR SIGHT ADDITION.—Two Numbers.—Add the numbers in rows 1 and 2, then in 2 and 3, then in 3 and 4, etc.

Thus, under column A, the numbers are 4 and 2, and their sum is 6; under column B the numbers are 5 and 7, and their sum is 12, etc.

Three Numbers.—Add the numbers in rows 1, 2, and 3, then in 2, 3, and 4, etc., in each column.

2. ADDITION OF SINGLE COLUMNS.—Any portion or the whole of any column may be used for this purpose.

3. ADDITION OF SEVERAL COLUMNS.—Two Columns.—Take the figures opposite 1, 2, and 3, in columns A and B, and find the sum; then those opposite 2, 3, and 4, in the same columns, and so on with all the rows and all the columns. Thus:

45	27	32	59	22
27	32	59	18	71
32	59	18	64	46

Three Columns.—Use numbers from columns A, B, and C; then from the other columns, as before. Thus:

452	278	324	591	186
278	324	591	186	645
324	591	186	645	227
591	186	645	227	712

More than Three Columns may be formed in a manner similar to that already given.

QUESTIONS FOR REVIEW.

55. What is addition? What is the sum or amount? What is the sign of addition? What is it called? Where is it placed? What is the sign of equality? How is it used? What is an equation? Give an example of one. What are the principles of addition? What are like numbers? What is the rule for addition? How is addition proved?

What is a unit? What is a number? What is arithmetic? What is numeration? What is notation? What is the Arabic system of notation? How many figures are there in a period? How many orders of units are there in each denomination? What are the names of the first five periods?

How are cents and mills written? How are the cents expressed if the number is less than ten?

What is Roman notation? What are the principles of Roman notation?

SUBTRACTION

INDUCTIVE EXERCISES.

56. 1. James had 4 apples, but he ate 2 of them. How many had he left?

2. Mr. Dane had 5 horses, but sold 3 of them. How many had he left?

3. A hen hatched 6 chickens, but 3 of them died. How many lived?

4. A boy had 7 cents, of which he spent 4. How many had he left?

5. A man earned \$10 per week, and spent \$5 of it. How much had he left?

6. Nine men fell from a scaffold, and 4 of them were killed. How many were not killed?

7. A freight train of 10 cars contained 4 open cars, and the rest were closed cars. How many closed cars were there in the train?

8. William bought a slate for 10 cents and sold it for 7 cents. How much did he lose?

9. In a certain class there are 9 girls and 5 boys. How many more girls are there than boys?

10. George is 11 years of age and Henry is 6. What is the difference between their ages? What is the difference between 11 and 6? Between 10 and 5?

11. What is the difference between 6 dollars and 3 dollars? Between 6 men and 3 dollars?

12. Why can you not find the difference between 6 men and 3 dollars?

13. Between what kind of numbers can you find the difference?

14. Hiram worked 8 days and William worked 5 days. How many days more did Hiram work than William?

15. Henry spent 12 cents and Jane spent 8 cents. How much more did Henry spend than Jane? What is the difference between 12 and 8?

16. If the difference between 12 and 8 is added to 8, the smaller number, what will be the result?

17. If the difference between any two numbers is added to the smaller number, to what will the result be equal?

DEFINITIONS.

57. **Subtraction** is the process of taking one number from another.

58. The **Minuend** is the number from which another is to be subtracted.

59. The **Subtrahend** is the number to be subtracted.

60. The **Remainder**, or **Difference**, is the result obtained by subtracting.

61. The **Sign of Subtraction** is a short horizontal line:
— . It is called *minus*.

When it is placed between two numbers it shows that the one after it is to be subtracted from the one before it.

Thus, $8-3$ is read 8 minus 3, and means that 3 is to be subtracted from 8.

62. **PRINCIPLES.**—1. *Only like numbers can be subtracted.*

2. *The sum of the subtrahend and remainder must be equal to the minuend.*

TABLE.

1-1= 0	2-2= 0	3-3= 0	4-4= 0	5-5= 0
2-1= 1	3-2= 1	4-3= 1	5-4= 1	6-5= 1
3-1= 2	4-2= 2	5-3= 2	6-4= 2	7-5= 2
4-1= 3	5-2= 3	6-3= 3	7-4= 3	8-5= 3
5-1= 4	6-2= 4	7-3= 4	8-4= 4	9-5= 4
6-1= 5	7-2= 5	8-3= 5	9-4= 5	10-5= 5
7-1= 6	8-2= 6	9-3= 6	10-4= 6	11-5= 6
8-1= 7	9-2= 7	10-3= 7	11-4= 7	12-5= 7
9-1= 8	10-2= 8	11-3= 8	12-4= 8	13-5= 8
10-1= 9	11-2= 9	12-3= 9	13-4= 9	14-5= 9
11-1=10	12-2=10	13-3=10	14-4=10	15-5=10
6-6= 0	7-7= 0	8-8= 0	9-9= 0	10-10= 0
7-6= 1	8-7= 1	9-8= 1	10-9= 1	11-10= 1
8-6= 2	9-7= 2	10-8= 2	11-9= 2	12-10= 2
9-6= 3	10-7= 3	11-8= 3	12-9= 3	13-10= 3
10-6= 4	11-7= 4	12-8= 4	13-9= 4	14-10= 4
11-6= 5	12-7= 5	13-8= 5	14-9= 5	15-10= 5
12-6= 6	13-7= 6	14-8= 6	15-9= 6	16-10= 6
13-6= 7	14-7= 7	15-8= 7	16-9= 7	17-10= 7
14-6= 8	15-7= 8	16-8= 8	17-9= 8	18-10= 8
15-6= 9	16-7= 9	17-8= 9	18-9= 9	19-10= 9
16-6=10	17-7=10	18-8=10	19-9=10	20-10=10

DRILL EXERCISES.

From 9 8 5 10 11 13 15 14 9 8 9 19 17 12
 Subtract 5 6 2 9 4 6 5 7 3 6 5 9 8 7

From 12 13 14 15 10 9 7 12 11 18 17 18 14 12
 Subtract 8 5 6 8 7 8 3 4 5 9 8 8 7 6

ORAL EXERCISES.

63. 1. I bought a writing-book for 15 cents and a slate for 10 cents. How much more did the writing-book cost than the slate?

2. A boy solved 12 examples on Monday and 15 on Tuesday. How many more did he solve on Tuesday than on Monday?

3. A horse traveled 9 miles the first hour and 7 miles the second. How much farther did he travel the first hour than the second?

4. Henry earned 14 cents and spent 7 of them. How much had he left?

5. A merchant has a piece of cloth containing 16 yards. If he sells 8 yards, how much will he have left?

6. A farmer who had 16 bushels of seed corn planted 7 bushels of it. How much had he left?

7. A boy who had 13 marbles lost eight of them. How many had he left?

8. William hoed 15 rows of corn while Henry hoed 7. How many more rows did William hoe than Henry?

9. I bought 16 pencils and sold 8 of them. How many had I left?

10. Henry's mother found that in 15 days at school he had perfect lessons on 7 days. On how many days did he have imperfect lessons?

11. A librarian loaned 14 books, of which all but 6 were story-books. How many story-books did he loan?

12. A news-boy who had purchased 17 papers sold all but 5. How many did he sell?

13. Two boys together caught 17 fishes. If one of them caught 8, how many did the other catch?

14. Fourteen sailors were wrecked and 5 of the number were drowned. How many were saved?

15. Seventeen criminals escaped from a jail, but 8 of them were caught. How many were not caught?

16. In a city there are 18 newspapers published, all but 6 of which are weeklies. How many weekly papers are published in that city?

17. A woman had 20 rose-bushes, but only 7 of them bloomed. How many did not bloom?

WRITTEN EXERCISES.

64. When no figure of the subtrahend has a greater value than the corresponding figure of the minuend.

1. From 869 subtract 423.

<p>PROCESS.</p> <p>Minuend 8 6 9</p> <p>Subtrahend <u>4 2 3</u></p> <p>Remainder <u>4 4 6</u></p>	<p>ANALYSIS.—For convenience the less number is written under the greater, units under units, tens under tens, etc.</p> <p>Beginning at the right, each order of units in the subtrahend is subtracted separately from the same order in the minuend.</p>
---	--

Thus, 9 units—3 units are 6 units, which are written under the units.
6 tens—2 tens are 4 tens, which are written under the tens.

8 hundreds—4 hundreds are 4 hundreds, which are written under the hundreds. Hence the remainder is 446.

PROOF.—446, the remainder, plus 423, the subtrahend, equals 869, the minuend. Hence the result is correct. (Prin. 2.)

Copy, subtract, and prove:

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
435	861	543	789	893	418
<u>214</u>	<u>320</u>	<u>212</u>	<u>345</u>	<u>371</u>	<u>217</u>
(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
864	957	329	899	564	888
<u>231</u>	<u>516</u>	<u>115</u>	<u>486</u>	<u>213</u>	<u>356</u>

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
2865	8793	5864	8694	7295	8346
<u>1523</u>	<u>4521</u>	<u>2651</u>	<u>5143</u>	<u>4052</u>	<u>1235</u>
(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
\$53.26	\$48.39	\$39.94	\$48.99	\$87.57	\$98.76
<u>31.15</u>	<u>24.15</u>	<u>22.71</u>	<u>35.43</u>	<u>51.43</u>	<u>35.41</u>

26. A man bought a farm for \$3685.29 and sold it for \$4796.39. How much did he gain?

27. A man purchased 8983 bricks, but used only 5362. How many had he left?

28. A merchant bought 30 pieces of cloth, containing in all 979 yards. He afterwards sold all but 143 yards. How many yards did he sell?

29. A man paid \$465 for a horse and carriage. If the carriage was worth \$215, how much was the horse worth?

30. A man purchased a city lot for \$1975, and afterwards sold it at a loss of \$315. How much did he get for it?

31. A man borrowed \$5694 without interest. If he paid \$3271, how much was still due?

32. A horse and cow together cost \$276. If the cow cost \$61, what was the cost of the horse?

33. A clerk earned in a year \$897.35, and spent \$635.20. How much did he save?

34. The cost of building a house was \$3972.59, and of furnishing it \$1561.30. How much more did it cost to build it, than to furnish it?

35. The population of a town in 1870 was 7693, and in 1880 it was 9796. How great was the increase?

36. A brigade of soldiers, upon going into battle, numbered 5864 men. After the engagement the number of men who answered the roll-call was 2733. How many were missing?

ORAL EXERCISES.

65. 1. Subtract by 4's from 20. Thus: 20, 16, 12, 8, etc.
2. Subtract by 3's from 30. By 4's from 35.
3. Subtract by 5's from 50. By 5's from 36.
4. Subtract by 4's from 36. By 6's from 36.
5. Subtract by 6's from 52. By 7's from 49.
6. Subtract by 8's from 64. By 9's from 99.
7. Subtract by 7's from 62. By 8's from 73.
8. How many are 25 less 5? 35 less 5? 45 less 5?
9. How many are 42 less 4? 52 less 4? 62 less 4?
10. How many are 36 less 7? 26 less 7? 46 less 7?
11. How many are 29 less 8? 59 less 8? 89 less 8?
12. How many are 31 less 6? 71 less 6? 61 less 6?
13. How many are 33 less 8? 53 less 8? 93 less 8?
14. If I owe a lad 8 cents, and have in my pocket 2 ten-cent pieces and 3 cents, what must I do before I can pay him?
15. After I get one of the ten-cent pieces changed into cents, how many cents will I have then?
16. When I pay him the 8 cents, how many cents will I have left? How many ten-cent pieces?
17. If I had 2 packages containing 10 pencils each and 4 pencils besides, what must I do to give a girl 7 pencils?
18. If I wish to take 4 things from 2 groups of 10 things and 2 things besides, what must be done to the groups containing 10 things?
19. A man had two stables containing 10 cows each and 4 cows besides. If he sells 8 cows, how many stables will there be which contain 10 cows? How many cows besides?
20. A boy piled his blocks in 3 piles of 10 each, and had 4 blocks besides. If he wishes to use 9 of the blocks, how many piles of 10 blocks each can he have left? How many blocks besides?

21. In taking 9 from 3 tens and 4 units, what is done with one of the tens? How many tens are left? How many units?

22. In taking 7 from 5 tens and 1 unit, what is done with one of the tens? How many tens are left? How many units?

23. In taking 6 from 4 tens and 3 units, or 43, what is done with one of the tens? How many tens are left? How many units?

24. How many tens and how many units are left when 7 is subtracted from 35?

WRITTEN EXERCISES.

66. When any figure of the subtrahend has a greater value than the corresponding figure of the minuend.

1. From 823 subtract 456.

PROCESS.		ANALYSIS.—For convenience the less number is written under the greater, units under units, tens under tens, etc.
Minuend	823	
Subtrahend	<u>456</u>	
Remainder	367	
		Beginning at the right, each order of units of the subtrahend is subtracted from the corresponding order in the minuend.

Since 6 units can not be subtracted from 3 units, 1 ten is united with the 3 units, making 13 units. 6 units from 13 units leave 7 units. The 7 is written in units' place in the remainder.

Since 1 ten was united with the units, there is but 1 ten left. Since 5 tens can not be subtracted from 1 ten, 1 hundred is united with the 1 ten, making 11 tens. 5 tens from 11 tens leave 6 tens. The 6 is written in tens' place in the remainder.

Since 1 hundred was united with the tens, there are but 7 hundreds left. 4 hundreds from 7 hundreds leave 3 hundreds. The 3 is written in hundreds' place in the remainder.

Hence the remainder is 367.

PROOF.—367, the remainder, plus 456, the subtrahend, equals 823, the minuend. Hence the result is correct. (Prin. 2.)

RULE.—Write the subtrahend under the minuend, units under units, tens under tens, etc.

Begin at the right and subtract each figure of the subtrahend from the corresponding figure of the minuend, writing the result beneath.

If a figure of the minuend has a less value than the corresponding figure of the subtrahend, increase the former by ten and subtract; then diminish by one the units of the next higher order in the minuend, and subtract as before.

PROOF.—Add together the remainder and the subtrahend. If the result is equal to the minuend, the work is correct.

Copy, subtract, and prove:

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
435	681	584	614	827	481	382
<u>266</u>	<u>479</u>	<u>293</u>	<u>285</u>	<u>563</u>	<u>194</u>	<u>267</u>
(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)
\$33.24	\$24.18	\$31.24	\$38.31	\$23.81	\$68.14	\$93.26
<u>12.85</u>	<u>16.26</u>	<u>14.76</u>	<u>17.26</u>	<u>14.56</u>	<u>32.76</u>	<u>47.35</u>

16. From 3000 subtract 1826.

PROCESS.	EXPLANATION.
Minuend 3000	Since 6 units can not be subtracted from 0 units, and there are no tens or hundreds, we change 1 thousand into hundreds, leaving 2 thousands, and 1 hundred into tens, leaving 9 hundreds, and 1 ten into units, leaving 9 tens. We now have 2 thousands, 9 hundreds, 9 tens, and 10 units, from which we can readily subtract the units of the subtrahend.
Subtrahend 1826	
Remainder 1174	

(17.)	(18.)	(19.)	(20.)	(21.)
2406	3608	3000	4000	60000
<u>1453</u>	<u>1567</u>	<u>1454</u>	<u>2683</u>	<u>23456</u>

Find the difference between :

- | | |
|--------------------|-------------------------|
| 22. 684 and 393. | 29. 4856 and 3004. |
| 23. 796 and 289. | 30. 5003 and 2675. |
| 24. 843 and 456. | 31. 48004 and 3865. |
| 25. 995 and 843. | 32. 27040 and 2568. |
| 26. 9256 and 7265. | 33. 470565 and 30465. |
| 27. 8413 and 3527. | 34. 380573 and 29683. |
| 28. 4568 and 3065. | 35. 4500640 and 320465. |

36. A city had, in 1882, 47265 inhabitants, which number was 4684 more than it had the previous year. How many had it the previous year?

37. The sales at a store in 1881 were \$36465, and in 1880 \$32694. How much more were they in 1880 than in 1881?

38. A farm was sold for \$9645.27, which was a gain of \$1685.19. How much did it cost?

39. A lumber merchant, having 38694 feet of a certain kind of lumber, sold 25897 feet of it. How much had he left?

40. A builder built a house for Mr. A. for \$8695.50. If Mr. A. has paid him but \$3869.75, how much does he still owe?

41. One number is 18695, and another is 5689 less than it. What is the other number?

42. A new church cost \$36845, which was \$18295 more than the old one cost. What did the old one cost?

43. A drover purchased 18694 cattle during the year, and sold them all, except 385 that he lost by accident and disease. How many did he sell?

44. A post-office sold 38695 stamps in 1881, which number was 3984 more than was sold the year previous. How many were sold in 1880?

45. The receipts of a manufactory were \$29756.83 and the expenses \$25462.79. What were the profits?

46. A vessel containing 16847 bushels of wheat leaked and damaged the cargo so that only 14329 bushels were fit for market. How many bushels were damaged?

REVIEW.

ORAL EXERCISES.

67. 1. $3 + 2 - 4 + 3 + 5 + 9 - 2 - 4 =$ how many?
2. $6 - 1 + 3 - 4 + 6 - 2 - 7 + 8 - 3 + 4 + 7 = ?$
3. $8 + 2 - 4 + 6 - 3 + 2 - 5 + 7 - 3 + 6 - 2 = ?$
4. $5 + 4 - 2 - 3 + 6 - 2 + 4 - 4 + 6 - 5 + 7 = ?$
5. $3 + 6 - 2 + 7 - 5 + 6 - 8 + 5 - 4 + 9 - 7 = ?$
6. $8 + 5 - 4 + 6 - 3 - 2 + 6 + 4 - 4 - 3 + 4 = ?$
7. $7 + 2 + 3 - 6 + 4 - 5 + 2 - 7 + 3 + 6 + 4 = ?$
8. $8 + 2 - 6 - 3 + 6 - 2 + 4 - 5 + 7 - 6 + 3 = ?$
9. $8 + 3 - 5 + 6 + 7 - 3 + 6 - 3 - 7 + 2 + 5 = ?$
10. $5 - 4 + 8 - 5 + 6 + 3 - 4 + 9 - 6 - 4 - 1 = ?$
11. $8 + 2 - 6 + 3 - 2 - 1 + 8 - 4 - 3 + 7 + 2 = ?$
12. $9 - 3 + 7 - 4 - 5 + 3 + 4 - 2 - 6 + 4 + 3 + 5 = ?$
13. Find the difference between the numbers in rows 1 and 2 in Drill Table, page 35. Then the difference between the numbers in rows 2 and 3, etc.
14. Find the difference between the numbers in row 3 and the sum of the numbers in rows 1 and 2. Then between the numbers in row 4 and the sum of the numbers in rows 2 and 3, etc.
15. Find the difference between the sum of the numbers in rows 1 and 2, and 3 and 4. Then between the sum of the numbers in rows 2 and 3, and 4 and 5, etc.
16. Henry, who had 15 cents, spent 5 cents for an orange, 4 cents for a pencil, and 3 cents for some paper. How much had he left?
17. Jane gave 12 gifts at Christmas. She gave her brothers

4, her sisters 2, her father 2, and the rest to her mother. How many did she give her mother?

18. There were 15 boys playing foot-ball, when 5 of them went home, and 4 left to play base-ball. Soon 3 new boys came and joined them. How many were there then playing foot-ball?

19. William owned 16 rabbits. He sold 4 to Thomas, 6 to Henry, and bought 3 from James. How many rabbits had he then?

20. At the beginning of the term, a class was composed of 16 pupils. Soon 4 left on account of sickness, 3 went to another school, and 5 new ones came. How many were there in the class then?

21. A grocer had 17 boxes of soap, of which he sold at one time 5 boxes, at another 3 boxes, at another 6 boxes. If he then bought 6 boxes, how many boxes had he?

22. A boy began to go to school when he was five years old. He spent 2 years at a kindergarten, 5 years at an academy, and the rest of the time, until he was 18 years of age, at the public schools. How long did he attend the public schools?

23. A street omnibus started with 10 passengers. During the trip 6 left it and 5 entered it. How many were on board at the end?

24. A girl had a twenty-five-cent piece and 5 cents. She purchased some ribbon for 8 cents, some braid for 5 cents, some candy for 3 cents. How much had she left?

25. A boy walked away from his home in one day 9 miles. He then stopped over night, and the next day walked 7 miles further. On the third day he turned and walked 8 miles towards home. How far from his home was he at the end of the third day?

26. James had 29 marbles and his uncle gave him 8 more. He sold 6, found 3, lost 7, and traded 5 to Henry for 4 of his. How many had he then?

WRITTEN EXERCISES.

68. 1. $7864 + 3297 - 4893 + 2164 =$ how many?
2. $3816 - 2938 + 4463 - 4156 =$ how many?
3. $8139 + 2685 - 4321 - 4126 =$ how many?
4. $13469 + 16846 - 3896 + 2164 =$ how many?
5. $26819 - 4869 - 13246 + 2684 =$ how many?
6. $34138 - 9876 - 8743 - 13412 =$ how many?
7. Three men purchased a building for \$36845. If the first paid \$10695, and the second \$8465, how much was there for the third to pay?
8. A man who was 23 years old in 1830, died at the age of 75. In what year did he die?
9. When a merchant began business he was worth \$8416. In the first year he gained \$3214, in the second he lost \$1629, and in the third he gained \$2147. How much was he worth then?
10. A man gave his eldest son \$3000, his second son \$375 less, and his third son \$625 less than the second. How much did all receive?
11. A man bought a farm for \$7216, expended \$315 repairing fences, \$1615 upon the buildings, and paid \$235 interest. If he sold the farm for \$10000, how much did he gain?
12. A merchant who had 394 yards of muslin, sold A 269 yards, and B the rest, lacking 27 yards. How many yards did he sell B?
13. A farmer who had 627 sheep, sold A 216, B 323, and 45 died. How many had he left?
14. A man wishing to purchase a horse worth \$200, has 3 twenty-dollar bills, 5 ten-dollar bills, and 6 five-dollar bills. If he borrows the rest, how much will he be obliged to borrow?
15. A merchant deposited in a bank the following sums:

on Monday, \$653.27; on Tuesday, \$423.18; on Wednesday, \$325.39. He drew out during that time \$629.84. How much did his deposits exceed what he drew out?

16. How much nearer \$5000 is \$6847 than \$2146?

17. A and B have each 1260 acres of land. If A sells to B 345 acres, how many has each then?

18. In a certain engagement the enemy began the battle with 23846 men. 8694 were wounded and 1463 killed. How many were fit for duty?

19. A farmer raised 3846 bushels of grain. He sold 496 bushels to A, 1864 bushels to B, and the rest to C. How many bushels did he sell to C?

20. The area of Massachusetts is 8315 square miles; of Pennsylvania, 45215 square miles; of Florida, 58680 square miles. How much larger is Florida than both the other States?

21. The expenses of a manufactory were \$8695.27 less than the receipts. If the receipts were \$18694.37, how much were the expenses?

22. A certain newspaper had a circulation of 113279 copies, and another 46396 copies. How much larger is the circulation of the first than that of the second?

23. Cotton was first planted in the United States about 1759. How long ago was that?

QUESTIONS FOR REVIEW.

69. What is subtraction? What is the minuend? What is the difference, or remainder? What is the sign of subtraction? When it is placed between two numbers, what does it show? What are the principles? Repeat the table. What is the rule for subtraction? What is addition? What are the principles of addition? What is notation? What is numeration? How many figures may there be in a period? How are cents written when the number is less than ten?

MULTIPLICATION

INDUCTIVE EXERCISES.

70. 1. Jane, Mary, and Sarah have each 3 books. How many books have they all?

2. I counted 3 piles of bricks each containing 3 bricks. How many bricks were there? How many would there be in 4 such piles?

3. I bought 3 pencils for 4 cents each. How much did they cost me?

4. There were 3 fields each containing 4 acres. How many acres were there in all the fields? How many are three 4's?

5. How many are four 4's? How many are five 4's?

6. A man earned \$4 per day. How much did he earn in 5 days?

7. An orchard contained 5 rows of 6 trees each. How many trees were there in the orchard?

8. How many are 5 times 6 trees? How many are 5 times 6?

9. James solved 5 problems per day for 6 days. How many problems did he solve?

10. How many problems are 6 times 5 problems? How many are 6 times 5?

11. How does 2 times 3 compare with 3 times 2? 6 times 5 with 5 times 6?

12. A man walked 4 miles per hour for 5 hours. How far did he walk in that time? How many are 5 times 4?

13. A dime is worth ten cents. How many cents are 6 dimes worth? How many are 6 times 10?

14. Table knives are often put up in sets of 6 knives. How many knives are there in 5 sets?

15. Five men each earned \$10 per week. How much did they all earn in one week?

When numbers are used without reference to any particular thing, they are called **Abstract Numbers**.

DEFINITIONS.

71. **Multiplication** is the process of repeating one number as many times as there are units in another; or,
A short process of finding the sum of equal numbers.

72. The **Multiplicand** is the number to be repeated or multiplied.

73. The **Multiplier** is the number showing how many times the multiplicand is to be repeated.

74. The **Product** is the result obtained by multiplying.

75. The multiplicand and multiplier are called the *factors* of the product.

76. The **Sign of Multiplication** is an oblique cross: \times . It is read, *multiplied by*, or *times*.

When it is placed between two numbers it shows that they are to be multiplied together.

Thus, 4×3 is read, 4 multiplied by 3, or 3 times 4.

77. **PRINCIPLES.**—1. *The multiplier must be regarded as an abstract number.*

2. *The multiplicand and product must be like numbers.*

3. *Either factor may be used as multiplicand or multiplier when both are abstract.*

TABLE.

$1 \times 1 = 1$	$2 \times 1 = 2$	$3 \times 1 = 3$	$4 \times 1 = 4$	$5 \times 1 = 5$
$1 \times 2 = 2$	$2 \times 2 = 4$	$3 \times 2 = 6$	$4 \times 2 = 8$	$5 \times 2 = 10$
$1 \times 3 = 3$	$2 \times 3 = 6$	$3 \times 3 = 9$	$4 \times 3 = 12$	$5 \times 3 = 15$
$1 \times 4 = 4$	$2 \times 4 = 8$	$3 \times 4 = 12$	$4 \times 4 = 16$	$5 \times 4 = 20$
$1 \times 5 = 5$	$2 \times 5 = 10$	$3 \times 5 = 15$	$4 \times 5 = 20$	$5 \times 5 = 25$
$1 \times 6 = 6$	$2 \times 6 = 12$	$3 \times 6 = 18$	$4 \times 6 = 24$	$5 \times 6 = 30$
$1 \times 7 = 7$	$2 \times 7 = 14$	$3 \times 7 = 21$	$4 \times 7 = 28$	$5 \times 7 = 35$
$1 \times 8 = 8$	$2 \times 8 = 16$	$3 \times 8 = 24$	$4 \times 8 = 32$	$5 \times 8 = 40$
$1 \times 9 = 9$	$2 \times 9 = 18$	$3 \times 9 = 27$	$4 \times 9 = 36$	$5 \times 9 = 45$
$1 \times 10 = 10$	$2 \times 10 = 20$	$3 \times 10 = 30$	$4 \times 10 = 40$	$5 \times 10 = 50$
$6 \times 1 = 6$	$7 \times 1 = 7$	$8 \times 1 = 8$	$9 \times 1 = 9$	$10 \times 1 = 10$
$6 \times 2 = 12$	$7 \times 2 = 14$	$8 \times 2 = 16$	$9 \times 2 = 18$	$10 \times 2 = 20$
$6 \times 3 = 18$	$7 \times 3 = 21$	$8 \times 3 = 24$	$9 \times 3 = 27$	$10 \times 3 = 30$
$6 \times 4 = 24$	$7 \times 4 = 28$	$8 \times 4 = 32$	$9 \times 4 = 36$	$10 \times 4 = 40$
$6 \times 5 = 30$	$7 \times 5 = 35$	$8 \times 5 = 40$	$9 \times 5 = 45$	$10 \times 5 = 50$
$6 \times 6 = 36$	$7 \times 6 = 42$	$8 \times 6 = 48$	$9 \times 6 = 54$	$10 \times 6 = 60$
$6 \times 7 = 42$	$7 \times 7 = 49$	$8 \times 7 = 56$	$9 \times 7 = 63$	$10 \times 7 = 70$
$6 \times 8 = 48$	$7 \times 8 = 56$	$8 \times 8 = 64$	$9 \times 8 = 72$	$10 \times 8 = 80$
$6 \times 9 = 54$	$7 \times 9 = 63$	$8 \times 9 = 72$	$9 \times 9 = 81$	$10 \times 9 = 90$
$6 \times 10 = 60$	$7 \times 10 = 70$	$8 \times 10 = 80$	$9 \times 10 = 90$	$10 \times 10 = 100$

DRILL EXERCISES.

Multiply	8	5	6	8	6	5	4	7	8	5	8	3	7
By	<u>4</u>	<u>3</u>	<u>7</u>	<u>5</u>	<u>9</u>	<u>8</u>	<u>6</u>	<u>9</u>	<u>4</u>	<u>7</u>	<u>6</u>	<u>9</u>	<u>5</u>

Multiply	6	7	5	4	7	6	5	4	7	4	5	7	8
By	<u>4</u>	<u>3</u>	<u>6</u>	<u>8</u>	<u>5</u>	<u>6</u>	<u>3</u>	<u>8</u>	<u>3</u>	<u>7</u>	<u>6</u>	<u>4</u>	<u>5</u>

For additional drill exercises see Drill Table, page 66.

ORAL EXERCISES.

78. 1. If a man walks 4 miles per hour, how far will he walk in 5 hours?

2. What will 5 slates cost at 8 cents apiece?

3. If a man makes 4 rods of fence per day, how many rods can he make in 6 days?

4. If the railroad fare from Auburn to Regent is \$4, how much will 5 tickets cost?

5. A man saved \$7 per week for 5 weeks. How much did he save?

6. Nine men each dug 4 rods of ditch. How many rods did they all dig?

7. If a boy earns \$7 per month, how much can he earn in 9 months?

8. If a man earns \$7 per week, and a boy \$3, how much will both earn in 7 weeks?

9. If 4 men can do a piece of work in 6 days, how long would it take 1 man to do it?

10. What will 8 quarts of berries cost at 10 cents per quart?

11. What will 9 yards of cloth cost at \$6 per yard?

12. What will 8 melons cost at 8 cents each?

13. If a man travels 8 hours at the rate of 5 miles per hour, how far does he travel?

14. How much must I pay for 9 sheep at \$7 per head?

15. A boy bought peaches at 8 cents per dozen. At that rate, what would 7 dozen cost?

16. A merchant sold 9 shawls at \$8 each. How much did he receive for them?

17. A news-boy's profits were but 9 cents per day for 5 days. How much did he make in that time?

18. Children are usually in school 6 hours per day. How many hours are they in school in 5 days?

19. A dime is 10 cents. How many cents are there in 8 dimes?

20. Repeat the table of 2's. Of 3's. Of 4's. Of 5's.

21. Repeat the table of 6's. Of 7's. Of 8's. Of 9's.
Of 10's.

22. Count by 3's from 0 to 30; thus, 0, 3, 6, 9, etc.

23. Count by 4's from 0 to 40. By 5's from 0 to 60.

24. Count by 6's from 0 to 72. By 7's from 0 to 84.

25. Count by 8's from 0 to 80. By 9's from 0 to 99.

26. Count by 2's from 0 to 100. By 10's from 0 to 100.

27. Count by 3's from 0 to 60. By 4's from 0 to 100.

WRITTEN EXERCISES.

79. When the multiplier is expressed by one figure.

1. How many are 4 times 365?

PROCESS.	ANALYSIS.—For convenience the multiplier
Multiplicand 365	is written under the multiplicand, and we begin
Multiplier 4	at the right to multiply.
Product 1460	4 times 5 units are 20 units, or 2 tens and
	0 units. The 0 is written in units' place
	in the product, and the 2 tens are reserved
	to add with the tens.

4 times 6 tens are 24 tens, plus 2 tens reserved are 26 tens, or 2 hundreds and 6 tens. The 6 tens are written in tens' place in the product, and the 2 hundreds reserved to add with the hundreds.

4 times 3 hundreds are 12 hundreds, plus 2 hundreds reserved are 14 hundreds, or 1 thousand and 4 hundreds, which are written in their proper places in the product.

Hence the product is 1460.

PROOF.—If the sum of four 365's is equal to 1460 the work is correct.

In multiplying, pronounce the results only. Thus, instead of saying 4 times 5 are 20, 4 times 6 are 24 and 2 reserved are 26, etc., say 20, 26, 14.

Solve and prove:

Solve:

Solve:

- | | | |
|------------------|-----------------------|------------------------|
| 2. 4 times 236. | 11. 4826×6 . | 20. 29613×5 . |
| 3. 5 times 246. | 12. 7834×5 . | 21. 31293×9 . |
| 4. 3 times 429. | 13. 5927×6 . | 22. 42617×6 . |
| 5. 3 times 826. | 14. 6984×7 . | 23. 32843×9 . |
| 6. 4 times 237. | 15. 5235×8 . | 24. 21697×8 . |
| 7. 2 times 894. | 16. 4296×7 . | 25. 31498×7 . |
| 8. 5 times 469. | 17. 2139×9 . | 26. 23426×9 . |
| 9. 4 times 832. | 18. 3128×6 . | 27. 42675×8 . |
| 10. 3 times 296. | 19. 4253×8 . | 28. 29376×7 . |

29. What will 9 pairs of shoes cost at \$4.25 a pair?

30. The wages of a mechanic were \$21.35 per week. How much did he earn in 6 weeks?

31. A man paid \$27.75 apiece for 5 sewing-machines. How much did they all cost him?

32. The average number of emigrants arriving at a city for 5 days was 3227 per day. How many arrived in that time?

33. A man's weekly expenses were \$27.63. How much did they amount to in 7 weeks?

34. A railroad company expended \$7265 in repairing each of 8 railway stations. How much was expended?

35. An army was composed of 8 brigades, each containing 6239 men. How many men were there in the army?

36. The daily edition of a newspaper for 9 days was 26475. How many papers were printed in that time?

37. There are 5280 feet in a mile. How many feet are there in 8 miles?

38. How much will 8 tons of hay cost at \$13.75 per ton?

39. A teamster drew 9 loads of bricks, each containing 1224 bricks. How many bricks did he draw?

40. A man bought 6 cows for \$42.35 per head. How much did he pay for them?

41. A man sold 6 acres of land for \$18.29 per acre. How much did he receive for it?

42. When coal is worth \$6.85 per ton, how much will 8 tons cost?

43. The annual expenses of a school for 6 years were \$6927. How much were the entire expenses for that time?

ORAL EXERCISES.

80. 1. How many are 10 times 5? 10 times 6? 10 times 7? 10 times 8? 10 times 4? 10 times 3?

2. What has to be annexed to 3 to obtain the product when 3 is multiplied by 10? What to 6? What to 4?

3. What has to be annexed to any number to obtain the product when the number is multiplied by 10?

4. How many are 10 times 15? 10 times 24? 10 times 35? 10 times 46? 10 times 37? 10 times 26?

5. How many are 10 times 6, plus 3 times 6, or 13 times 6?

6. How many are 10 times 4, plus 4 times 4, or 14 times 4?

7. How many are 10 times 7, plus 2 times 7, or 12 times 7?

8. How many are 14 times 5, or 10 times 5, plus 4 times 5?

9. How many are 13 times 8, or 10 times 8, plus 3 times 8?

10. How many are 15 times 5, or 10 times 5, plus 5 times 5?

11. How then may we multiply by 16? By 18? By 17? By 19? By 13? By 12? By 14? By 15?

12. How many are 16 times 5? 16 times 4?

13. How many are 13 times 5? 13 times 3?

14. How many are 15 times 4? 15 times 6?
15. What will 13 yards of broadcloth cost at \$4 per yard?
16. What will 15 pairs of skates cost at \$3 a pair?
17. What will 18 dolls cost at \$2 apiece?
18. If a cow gives 8 quarts of milk per day, how much will she give in 12 days?
19. If a cooper can make 7 barrels per day, how many can he make in 13 days?
20. If sugar is worth 9 cents a pound, what will 14 pounds cost?
21. If a man saves \$8 per month, how much will he save in 12 months?
22. What will be the cost of 13 pounds of cheese at 7 cents per pound?
23. What will be the cost of 15 yards of muslin at 8 cents per yard?
24. A farmer had 6 cows and 15 times as many sheep. How many sheep had he?
25. How do 10 nines compare with 9 tens? 100 nines with 9 hundreds? 1000 nines with 9 thousands?
26. How many then are 100 times 9? 100 times 8? 100 times 7?
27. What do we annex to a number when we multiply it by 100?
28. How many are 1000 times 9? 1000 times 8? 1000 times 7?
29. What do we annex to a number when we multiply it by 1000?
30. How many are 100 times 5? 100 times 6? 1000 times 5? 1000 times 6?

81. PRINCIPLE.—*A number is multiplied by 10, 100, 1000, etc., by annexing to the multiplicand as many ciphers as there are in the multiplier.*

WRITTEN EXERCISES.

82. When the Multiplier is expressed by more than one figure.

1. Multiply 218 by 1000.

PROCESS. ANALYSIS.—For convenience the first significant figure of the multiplier is written under the units' figure of the multiplicand. Then the product is obtained by annexing three ciphers to the multiplicand. (Prin., Art. 81.)

	(2.)	(3.)	(4.)	(5.)
Multiply	468	379	269	2650
By	<u>100</u>	<u>1000</u>	<u>100</u>	<u>1000</u>

	(6.)	(7.)	(8.)	(9.)
Multiply	2304	3400	3000	4600
By	<u>100</u>	<u>100</u>	<u>1000</u>	<u>1000</u>

10. Multiply 668 by 2000.

PROCESS. ANALYSIS.—Since 2000 is equal to 2 times 1000, we multiply 668 by 2, obtaining the product 1366, and then multiply the result by 1000 by annexing three ciphers.

	(11.)	(12.)	(13.)	(14.)	(15.)
Multiply	384	469	274	568	296
By	<u>20</u>	<u>30</u>	<u>50</u>	<u>70</u>	<u>40</u>

	(16.)	(17.)	(18.)	(19.)	(20.)
Multiply	596	497	568	461	823
By	<u>200</u>	<u>300</u>	<u>700</u>	<u>200</u>	<u>600</u>

21. Multiply 264 by 132.

FIRST PROCESS.

	264
	<u>132</u>
1st Partial Prod.	528
2d Partial Prod.	7920
3d Partial Prod.	<u>26400</u>
Entire Prod.	34848

7920, the *second* partial product; 1 hundred, or 100, times 264 equals 26400, the *third* partial product. The sum of these partial products is the *entire* product.

SECOND PROCESS.

	264
	<u>132</u>
1st Partial Prod.	528
2d Partial Prod.	792
3d Partial Prod.	<u>264</u>
Entire Prod.	34848

In multiplying by hundreds the lowest order of the product is hundreds; hence the first figure of the product is written under hundreds.

PROOF.—Multiply the multiplier by the multiplicand. (Prin. 3.) If the result agrees with that formerly obtained, the work is probably correct.

83. RULE.—*Write the multiplier under the multiplicand, units under units, tens under tens, etc.*

Multiply each figure of the multiplicand by each significant figure of the multiplier successively, beginning with units. Place the right-hand figure of each product under the figure of the multiplier used to obtain it, and add the partial products.

PROOF.—*Review the work, or multiply the multiplier by the multiplicand.*

ANALYSIS.—For convenience the multiplier is written under the multiplicand, units under units, tens under tens, etc.

Since we can not multiply by 132 at one operation, we multiply by the parts, 2, 3 tens, or 30, and 1 hundred, or 100, and add the products.

2 times 264 equals 528, the *first* partial product; 3 tens, or 30, times 264 equals 7920, the *second* partial product; 1 hundred, or 100, times 264 equals 26400, the *third* partial product. The sum of these partial products is the *entire* product.

Thus, in multiplying 264 by 2 *tens* the product is 792 *tens*, or 7 thousands, 9 hundreds, 2 tens, which are written in their proper places in the partial product.

In multiplying by hundreds the lowest

When there is a cipher in the multiplier, multiply by the significant figures only, taking care to place the first figure in the product under the figure of the multiplier used to obtain it.

$$\begin{array}{r}
 (1.) \\
 2693 \\
 307 \\
 \hline
 18851 \\
 8079 \\
 \hline
 826751
 \end{array}$$

$$\begin{array}{r}
 (2.) \\
 2465 \\
 2800 \\
 \hline
 19720 \\
 4930 \\
 \hline
 6902000
 \end{array}$$

$$\begin{array}{r}
 (3.) \\
 36954 \\
 4006 \\
 \hline
 221724 \\
 147816 \\
 \hline
 148037724
 \end{array}$$

Multiply:

4. 269 by 32.
5. 485 by 24.
6. 693 by 34.
7. 725 by 43.
8. 826 by 52.
9. 927 by 35.
10. 826 by 24.
11. 914 by 55.
12. 429 by 63.
13. 628 by 48.
14. 721 by 57.
15. 6843 by 67.
16. 8725 by 74.
17. 8629 by 93.
18. 1286 by 98.
19. 2794 by 69.
20. 2385 by 57.
21. 2684 by 36.

Multiply:

22. \$23.48 by 35.
23. \$16.29 by 67.
24. \$24.18 by 39.
25. \$24.35 by 43.
26. \$23.09 by 65.
27. \$48.31 by 68.
28. 29685 by 134.
29. 38964 by 305.
30. 29875 by 469.
31. 28576 by 431.
32. 29314 by 230.
33. 26857 by 424.
34. 68735 by 169.
35. 46853 by 218.
36. 468546 by 2134.
37. 678956 by 3045.
38. 897362 by 4606.
39. 812357 by 4328.

40. 684579 by 323500.

41. A railway engine traveled on an average 237 miles per day. How far did it travel in 31 days?

42. I sold my farm of 392 acres at \$105 per acre. How much did I get for it?

43. At a railway station an average of 436 tickets was sold daily for 312 days. How many tickets were sold?

44. The average daily expenses of a manufactory were \$8361. How much would they amount to in 126 days?

45. A man bought railroad stock for \$24.13 a share. How much did he pay for 223 shares?

46. A regiment of soldiers consumed 975 pounds of meat per day. How much would they consume in 200 days?

47. There are 480 sheets of paper in a ream. How many sheets were destroyed by a fire that consumed 246 reams?

48. A merchant purchased 25 pieces of broadcloth, each containing 48 yards, at \$7 per yard. How much did he pay for the whole?

49. If a steamboat goes 16 miles an hour, how far will it go in 7 days of 24 hours each?

50. If a man has 316 acres of wheat, which produce 27 bushels per acre, how many bushels will he get from the whole?

REVIEW.

ORAL EXERCISES.

84. 1. Bought 3 pencils at 4 cents each and 4 oranges at 5 cents each. How much did I pay for all?

2. Two men travel in opposite directions from a certain place, one at 5 miles per hour, and the other at 3 miles per hour. How far will they be apart at the end of 4 hours?

3. If the men travel in the same direction, how far will they be apart at the end of 10 hours?

4. If a man earns \$5 per week and a boy earns \$2 per

week, how much more will the man earn in 10 weeks than the boy?

5. A merchant sold 5 yards of cloth at \$4 per yard and 3 yards of lining at \$1 per yard. How much did he get for both?

6. If 12 teams can do a piece of work in 8 days, how long will it take one team to do the same work?

7. A man bought 4 calves at \$5 apiece and 3 sheep at \$4 apiece. How much did he pay for all?

8. Two brothers received each \$4 per week for their services. How much did both earn in 8 weeks?

9. If a man earns \$15 per week and spends \$10, how much will he save in 8 weeks?

10. James has \$5 in the bank and his brother lacks \$4 of having 4 times as much. How much has his brother?

11. A farmer bought 10 pounds of sugar at 8 cents per pound, and gave in payment 4 dozen eggs at 12 cents per dozen, and the rest in cash. How much did he pay in cash?

12. A man bought a plow for \$12 and 2 harrows for \$8 each. How much did he pay for all?

13. A miller bought wheat at \$1 a bushel. It took 20 bushels to make 4 barrels of flour. If he sold the flour at \$6 per barrel, how much did he gain on each 20 bushels?

14. Henry earned \$6 per week and James earned \$3. How much more did Henry earn than James in 10 weeks?

15. A man purchased 3 bunches of asparagus at 8 cents per bunch and 2 quarts of strawberries at 10 cents per quart. How much did both cost him?

16. Bought 3 pounds of beef at 12 cents per pound and 2 bunches of radishes at 5 cents a bunch. If I give a fifty-cent piece in payment, how much should I get in change?

17. Bought 3 quarts of milk at 6 cents a quart and 3 pounds of biscuits at 8 cents a pound. How much should I receive back if I gave in payment 50 cents?

18. After purchasing 4 yards of cambric at 10 cents a yard, and 3 spools of thread at 5 cents a spool, I had 15 cents left. How much money had I?

19. Bought 2 pounds of coffee at 35 cents a pound and 2 pounds of rice at 10 cents a pound. If I gave in payment one dollar, how much should I have returned to me?

20. Henry's expenses for books were 64 cents and for stationery 20 cents. How much change should he receive if he gave in payment one dollar?

WRITTEN EXERCISES.

85. 1. At a church collection one box contained \$18.50, another \$6 25, another \$23.18, and another \$13.29. How much did they all contain?

2. In an orchard there were 17 rows containing 24 trees each, but 18 of the whole number were dead. How many trees were alive?

3. Sold 320 tons of coal at \$8 per ton, and received in payment a house valued at \$1000, and the rest in cash. How much did I receive in cash?

4. A farmer bought 40 sheep at \$7 each, 40 cows at \$30 per head, and 4 horses at \$200 each. He sold them all for \$3000. How much did he gain?

5. What will be the cost of 213 chests of tea, each containing 52 pounds, at 40 cents a pound?

6. A drover sold 25 oxen at \$65 each, 214 sheep at \$4.50 per head, and 60 calves at \$3 each, gaining by the transaction \$200.24. How much did they cost him?

—7. I had a farm of 413 acres. 215 acres yielded me an income of \$4.50 per acre and the rest yielded me an income of \$7 per acre. How much was my entire income from the farm?

8. I sold 13 firkins of butter, each containing 83 pounds,

at 30 cents a pound, and gained \$10.15. How much did the butter cost me?

9. The railroad fare from A to B was \$1.86. How much would be received from 13 car-loads of passengers if each car held 50 passengers?

10. A grain merchant sold 200 bushels of corn for 63 cents per bushel, and lost by the sale \$8.94. How much did it cost him?

11. Two persons leave the same place at the same time and travel in the same direction, one at the rate of 35 miles per day, and the other 42 miles per day. How far will they be apart at the end of 45 days?

12. The expenses of a pleasure trip were \$8.73 for each of the 9 persons composing the party. If the entire contributions were \$60.24, how much must yet be raised?

13. A sale of stock resulted as follows: 9 cows were sold for \$36 each, 8 horses for \$136 apiece, and 65 sheep for \$4.75 per head. The cows cost \$35 per head, the horses \$140, and the sheep \$3. Did I gain or lose by the sale, and how much?

14. A's barn cost \$1865, his house cost 3 times as much, and his farm, exclusive of the house and barn, twice as much as both. If he accepts an offer of \$20000 for his property, does he gain or lose, and how much?

15. A and B each began business with \$4000. A gained \$500 the first year, and B lost an equal sum. The second year B gained \$645, and A gained \$595. The third year A gained \$1640, and B lost \$40. How much more than B has A at the end of the three years?

16. A wholesale clothier bought 693 coats at \$8.25 each. He afterward sold 510 of them at \$10 each and the rest at \$7.50 apiece. Did he gain or lose, and how much?

17. A merchant's profits from his business were \$8695. If he paid \$869 for house rent, and 3 times as much for other expenses, how much did he save?

2. **The multiplicand more than one figure.**—Take two or more figures or the whole of row 1 for a multiplicand, and for multipliers each of the numbers in row 2 successively, then two or more figures or the whole of row 2 for a multiplicand, and for multipliers each of the numbers in row 3 successively, etc. Thus:

6543	6543	65438
2	6	8
<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>

THE MULTIPLIER MORE THAN ONE FIGURE.—Use for a multiplicand any portion or the whole of row 1, and for multipliers the first two, three, or more figures, then the second two, three, or more figures in row 2, etc., proceeding as already directed. Thus:

6543	54384	54384
26	26	68
<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>
4384	43842	38429
684	843	4397
<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>

QUESTIONS FOR REVIEW.

87. What is multiplication? What is the multiplicand? What is the multiplier? What is the product? What are the factors of the product? What is the sign of multiplication? What does it show? How is it read? What are the principles of multiplication? Repeat the table? What is the principle in regard to multiplying by 10, 100, 1000, etc.? What is the rule for multiplication? How do we multiply when there is a cipher in the multiplier?

What is subtraction? What is the minuend? The subtrahend? The remainder? What is the rule for subtraction? What is the rule for notation? For numeration?



INDUCTIVE EXERCISES.

88. 1. How many groups of two blocks each can be made from 4 blocks? How many 2's are there in 4?

2. How many groups of 3 cents each can be formed from 6 cents? How many 3's are there in 6?

3. How many groups of 3 children each can be formed from 9 children? How many 3's are there in 9?

4. How many groups of 4 things can be formed from 8 things? How many 4's are there in 8?

5. Since there are 2 pints in a quart, how many quarts are there in 6 pints? How many times are 2 pints contained in 6 pints?

6. How many times can a man who has \$6 give some boys \$2? How many times are \$2 contained in \$6?

7. William had 8 cents, which he spent for candy, spending 2 cents at a time. How many times did he buy candy? How many times are 2 cents contained in 8 cents?

8. If you pay 2 cents for a pencil, how many can you buy for 10 cents? How many times are 2 cents contained in 10 cents?

9. If 12 trees are arranged in 4 equal rows, how many trees will there be in each row?

10. How many 2's are there in 6? In 8?

11. How many 3's are there in 6? In 9?

12. How many 4's are there in 8? In 12?

13. How many packages of 5 cards each can be made from 15 cards? How many 5's are there in 15?

14. A measure containing 2 quarts was used to dip 10 quarts of water out of a cask. How many times was the measure filled? How many times are 2 quarts contained in 10 quarts?

15. A boy arranged 12 books, putting 3 books in a pile. How many piles did he form? How many 3's are there in 12? How many times are 3 books contained in 12 books?

DEFINITIONS.

89. Division is the process of finding how many times one number is contained in another; or,

The process of separating a number into equal parts.

90. The Dividend is the number to be divided.

91. The Divisor is the number by which we divide.

92. The Quotient is the result obtained by division.

93. The Remainder is the part of the dividend remaining when the division is not exact.

94. The Sign of Division is \div . It is read *divided by*. When placed between two numbers it shows that the one at the left is to be divided by the one at the right.

Thus, $24 \div 6$ is read 24 divided by 6.

Division is also indicated by writing the dividend above the divisor, with a line between them.

Thus, $\overset{24}{\underset{6}{\div}}$ is read 24 divided by 6.

Another method of indicating division is by writing the divisor at the left of the dividend, with a line between them.

Thus, $6)24$ is read 24 divided by 6.

EQUAL PARTS.

95. When any thing is divided into *two* equal parts, each part is called *one-half*.

When any thing is divided into *three* equal parts, each part is called *one-third*; when into *four* equal parts, each part is called *one-fourth*; when into *five* equal parts, *one-fifth*; when into *twenty-five* equal parts, *one twenty-fifth*.

96. One or more of the equal parts of any thing are called a **Fraction**.

97. *Equal parts* of a number may be expressed by figures by writing the number denoting the name of the parts below a short horizontal line, and the number denoting how many are used, above the line. Thus,

3 of 5 equal parts, or *three-fifths*, are written $\frac{3}{5}$.

1 of 2 equal parts, or *one-half*, is written $\frac{1}{2}$.

1 of 3 equal parts, or *one-third*, is written $\frac{1}{3}$.

1 of 4 equal parts, or *one-fourth*, is written $\frac{1}{4}$.

2 of 3 equal parts, or *two-thirds*, are written $\frac{2}{3}$.

3 of 4 equal parts, or *three-fourths*, are written $\frac{3}{4}$.

7 of 8 equal parts, or *seven-eighths*, are written $\frac{7}{8}$.

5 of 13 equal parts, or *five-thirteenth*s, are written $\frac{5}{13}$.

EXERCISES.

98. 1. Read the expression $\frac{3}{17}$.

MODEL.— $\frac{3}{17}$ is read three seventeenths, or three of seventeen equal parts.

Read in a similar manner the following:

$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{7}$	$\frac{4}{9}$	$\frac{5}{13}$	$\frac{2}{6}$	$\frac{3}{8}$	$\frac{4}{7}$	$\frac{5}{9}$	$\frac{6}{13}$
$\frac{4}{28}$	$\frac{3}{37}$	$\frac{13}{25}$	$\frac{16}{64}$	$\frac{13}{40}$	$\frac{24}{37}$	$\frac{15}{34}$	$\frac{13}{29}$	$\frac{12}{36}$	$\frac{16}{64}$

TABLE.

$1 \div 1 = 1$	$2 \div 2 = 1$	$3 \div 3 = 1$	$4 \div 4 = 1$	$5 \div 5 = 1$
$2 \div 1 = 2$	$4 \div 2 = 2$	$6 \div 3 = 2$	$8 \div 4 = 2$	$10 \div 5 = 2$
$3 \div 1 = 3$	$6 \div 2 = 3$	$9 \div 3 = 3$	$12 \div 4 = 3$	$15 \div 5 = 3$
$4 \div 1 = 4$	$8 \div 2 = 4$	$12 \div 3 = 4$	$16 \div 4 = 4$	$20 \div 5 = 4$
$5 \div 1 = 5$	$10 \div 2 = 5$	$15 \div 3 = 5$	$20 \div 4 = 5$	$25 \div 5 = 5$
$6 \div 1 = 6$	$12 \div 2 = 6$	$18 \div 3 = 6$	$24 \div 4 = 6$	$30 \div 5 = 6$
$7 \div 1 = 7$	$14 \div 2 = 7$	$21 \div 3 = 7$	$28 \div 4 = 7$	$35 \div 5 = 7$
$8 \div 1 = 8$	$16 \div 2 = 8$	$24 \div 3 = 8$	$32 \div 4 = 8$	$40 \div 5 = 8$
$9 \div 1 = 9$	$18 \div 2 = 9$	$27 \div 3 = 9$	$36 \div 4 = 9$	$45 \div 5 = 9$
$10 \div 1 = 10$	$20 \div 2 = 10$	$30 \div 3 = 10$	$40 \div 4 = 10$	$50 \div 5 = 10$
$6 \div 6 = 1$	$7 \div 7 = 1$	$8 \div 8 = 1$	$9 \div 9 = 1$	$10 \div 10 = 1$
$12 \div 6 = 2$	$14 \div 7 = 2$	$16 \div 8 = 2$	$18 \div 9 = 2$	$20 \div 10 = 2$
$18 \div 6 = 3$	$21 \div 7 = 3$	$24 \div 8 = 3$	$27 \div 9 = 3$	$30 \div 10 = 3$
$24 \div 6 = 4$	$28 \div 7 = 4$	$32 \div 8 = 4$	$36 \div 9 = 4$	$40 \div 10 = 4$
$30 \div 6 = 5$	$35 \div 7 = 5$	$40 \div 8 = 5$	$45 \div 9 = 5$	$50 \div 10 = 5$
$36 \div 6 = 6$	$42 \div 7 = 6$	$48 \div 8 = 6$	$54 \div 9 = 6$	$60 \div 10 = 6$
$42 \div 6 = 7$	$49 \div 7 = 7$	$56 \div 8 = 7$	$63 \div 9 = 7$	$70 \div 10 = 7$
$48 \div 6 = 8$	$56 \div 7 = 8$	$64 \div 8 = 8$	$72 \div 9 = 8$	$80 \div 10 = 8$
$54 \div 6 = 9$	$63 \div 7 = 9$	$72 \div 8 = 9$	$81 \div 9 = 9$	$90 \div 10 = 9$
$60 \div 6 = 10$	$70 \div 7 = 10$	$80 \div 8 = 10$	$90 \div 9 = 10$	$100 \div 10 = 10$

DRILL EXERCISES.

$8 \div 2$	$12 \div 3$	$16 \div 4$	$18 \div 3$	$28 \div 7$	$42 \div 6$	$36 \div 6$
$21 \div 3$	$45 \div 9$	$63 \div 7$	$28 \div 4$	$40 \div 8$	$48 \div 8$	$32 \div 4$
$63 \div 9$	$35 \div 7$	$45 \div 5$	$30 \div 5$	$27 \div 9$	$32 \div 8$	$40 \div 4$
$49 \div 7$	$56 \div 7$	$48 \div 6$	$42 \div 7$	$54 \div 9$	$81 \div 9$	$36 \div 4$
$\frac{42}{7}$	$\frac{32}{8}$	$\frac{64}{8}$	$\frac{70}{10}$	$\frac{35}{5}$	$\frac{42}{7}$	$\frac{24}{3}$
$\frac{36}{6}$	$\frac{80}{10}$	$\frac{36}{4}$	$\frac{64}{8}$	$\frac{72}{9}$	$\frac{45}{9}$	$\frac{30}{6}$
$\frac{56}{8}$	$\frac{45}{5}$	$\frac{63}{9}$	$\frac{72}{8}$	$\frac{81}{9}$	$\frac{32}{8}$	$\frac{40}{10}$
$\frac{32}{8}$	$\frac{45}{5}$	$\frac{63}{9}$	$\frac{72}{8}$	$\frac{81}{9}$	$\frac{32}{8}$	$\frac{40}{10}$

ORAL EXERCISES.

99. 1. If a man earns \$3 per day, how long will it take him to earn \$12?

2. At 5 cents a quart, how many quarts of milk can be bought for 25 cents?

3. At 10 cents per quire, how many quires of paper can be bought for 50 cents?

4. If a boy spends 5 cents a day, in how many days will he spend 45 cents?

5. There are 7 days in a week. How many weeks are there in 49 days?

6. At 4 cents per pound, how many pounds of nails can be bought for 36 cents?

7. If coal costs \$7 per ton, how many tons can be bought for \$49?

8. If muslin is worth 8 cents per yard, how many yards can be bought for 72 cents?

9. How many bicycles at \$9 apiece can be bought for \$81?

10. At \$5 per head, how many sheep can be bought for \$45?

11. If a horse can travel 6 miles per hour continuously, in how many hours can he travel 54 miles?

12. If a certain kind of coat is worth \$8, how many such can be bought for \$56?

13. If 42 apples were divided among some boys by giving each 7 apples, how many boys were there?

14. There are 4 quarts in a gallon. How many gallons are there in 32 quarts?

15. How many blocks 6 inches in length can be cut from a board 54 inches long?

16. An agent received \$45 for books which he sold at \$5 each. How many books did he sell?

17. If a farmer receives a dollar for 4 pounds of butter, how many dollars will he receive for 36 pounds?

18. A man who had 48 acres of land divided it into 6 fields of equal size. What part of the whole was each field? How many acres were there in each?

19. James sold 40 marbles to 5 boys, selling the same number to each. How many did he sell to each? What part of the whole did he sell to each?

20. A man sold 40 gallons of kerosene in 5 hours. How much did he sell per hour?

21. A hound ran 60 miles in 6 hours. How far did he run per hour?

22. A cow ate 63 pounds of mill-feed in 7 days. How many pounds did she eat per day?

23. If a man has \$37, how many sheep at \$5 per head can he buy with the money? How much money will he have left?

100. PRINCIPLE.—*The product of the divisor and quotient, plus the remainder, is equal to the dividend.*

WRITTEN EXERCISES.

101. When the divisor is expressed by one figure.

1. Divide 1701 by 3.

PROCESS.

Divisor. Dividend. Quotient.

3) 1701 (567

15 h. t. u.

20

18

21

21

hundreds are 17 hundreds. 3 is contained in 17 hundreds 5 hun-

ANALYSIS.—For convenience the divisor is written at the left, and the quotient at the right of the dividend.

We begin at the left to divide. Thus, 3 is not contained in 1 thousand any thousand times, therefore the quotient can not contain units of any order higher than hundreds. Hence we find how many times 3 is contained in all the hundreds of the dividend. 1 thousand plus 7

dred times and a remainder. We write the 5 in hundreds' place in the quotient, and multiply the divisor by it, obtaining for a product 15 hundreds, or 1 thousand and 5 hundreds, which we write under units of the same order in the dividend. Subtracting this product from the partial dividend, there is a remainder of 2 hundreds.

2 hundreds plus 0 tens are 20 tens. 3 is contained in 20 tens, 6 tens times and a remainder. We write the 6 tens in tens' place in the quotient, and multiply the divisor by it, obtaining for a product 18 tens, or 1 hundred and 8 tens, which we write under units of the same order in the partial dividend. Subtracting, there is a remainder of 2 tens.

2 tens plus 1 unit are 21 units. 3 is contained in 21 units 7 times. We write the 7 units in units' place in the quotient, and multiply the divisor by it, obtaining for a product 21, or 2 tens, and 1 unit, which we write under units of the same order in the partial dividend. Subtracting, there is no remainder.

Hence the quotient is 567.

PROOF.—567, the quotient, multiplied by 3, the divisor, gives as a product 1701. Hence the work is correct. (Prin., Art. 100.)

Solve in like manner and prove:

- | | | |
|--------------------|--------------------|---------------------|
| 2. $852 \div 3$. | 6. $2124 \div 4$. | 10. $1395 \div 5$. |
| 3. $2562 \div 6$. | 7. $1191 \div 3$. | 11. $1340 \div 4$. |
| 4. $2135 \div 5$. | 8. $1584 \div 6$. | 12. $2142 \div 6$. |
| 5. $1603 \div 7$. | 9. $2345 \div 5$. | 13. $1413 \div 3$. |

102. When examples in division are solved without writing the products or remainders, the process is called **Short Division**.

14. Divide 4821 by 3.

PROCESS. ANALYSIS.—3 is contained in 4 thousand 1 thousand times and 1 thousand remainder. We write the 1 in the quotient in thousands' place.

$$\begin{array}{r} 3 \overline{) 4821} \\ \underline{1607} \end{array}$$

1 thousand remainder and 8 hundreds are 18 hundreds. 3 is contained in 18 hundreds 6 hundred times. We write the six in hundreds' place in the quotient.

3 is not contained in 2 tens any tens times; hence we write 0 in tens' place in the quotient.

2 tens and 1 unit are 21 units. 3 is contained in 21 units 7 times. We write the 7 in units' place in the quotient.

Hence the quotient is 1607.

Divide by *short division*:

15. $4872 \div 4$.	23. $\begin{array}{r} 4986 \\ 9 \end{array}$.	31. $84364 \div 7$.
16. $6830 \div 5$.	24. $\begin{array}{r} 6765 \\ 5 \end{array}$.	32. $38857 \div 8$.
17. $2976 \div 6$.	25. $\begin{array}{r} 3836 \\ 7 \end{array}$.	33. $\$24.35 \div 5$.
18. $2985 \div 5$.	26. $\begin{array}{r} 3872 \\ 8 \end{array}$.	34. $\$13.62 \div 3$.
19. $4635 \div 3$.	27. $\begin{array}{r} 4567 \\ 5 \end{array}$.	35. $\$14.22 \div 6$.
20. $3936 \div 4$.	28. $\begin{array}{r} 3932 \\ 6 \end{array}$.	36. $\$27.37 \div 7$.
21. $3248 \div 8$.	29. $\begin{array}{r} 3174 \\ 9 \end{array}$.	37. $\$18.36 \div 4$.
22. $5256 \div 6$.	30. $\begin{array}{r} 3185 \\ 4 \end{array}$.	38. $\$24.24 \div 8$.

39. How many plows at \$8 each can be bought for \$1328?

40. How many coats at \$5 each can be bought for \$1125?

41. If a ship sails 8 miles per hour, how long will it take her to sail 1248 miles?

42. If a man spends \$7 per week, how long will it take him to spend \$1477?

43. The aggregate salaries of 5 men are \$3765. If each receives the same sum, what is the salary of each?

44. A stage traveled 6 miles per hour. How many hours would it require to travel 2352 miles?

45. The traveling expenses of an agent averaged \$5 per day, and his entire traveling expenses were \$1435. How many days did he travel?

46. At a church collection each person contributed on the average 7 cents. If the entire sum collected was 4823 cents, how many persons contributed?

47. A church cost 7 times as much as the organ. If the church cost \$12355, how much did the organ cost?

48. The annual expenses of 5 schools were \$28695. If

the expenses were alike for all, how much were the expenses of each?

49. How many yards of cloth at 9 cents per yard can be bought for 29439 cents?

ORAL EXERCISES.

103. 1. At 10 cents a yard, how many yards of cambrio can be bought for 40 cents?

2. At 12 cents a pound, how many pounds of sugar can be bought for 36 cents?

3. There are 25 cents in a quarter of a dollar. To how many quarter-dollars are 50 cents equal?

4. If I pay \$60 for 20 lambs, how much do I pay for each?

5. A real estate dealer divided 30 acres into 15 equal lots. How many acres were there in each?

6. If a man earns \$11 per week, how long will it take him to earn \$44?

7. There are 24 hours in a day. If a man walks 72 hours, how many days does he walk?

8. If I sell books at 30 cents each, how many must I sell to receive 90 cents?

9. If a news-boy sells 40 papers per day, in how many days will he sell 120 papers?

10. Twelve articles make a dozen. How many dozen are there in 24 articles? In 60?

11. Twenty-four sheets make a quire of paper. How many quires are there in 48 sheets? In 72?

12. How many are 10 times 6? 10 times 9? 10 times 12? 10 times 25? 10 times 35? 10 times 48?

13. How many 10's are there in 60? In 90? In 120? In 250? In 350? In 480?

14. Since in 60 there are 6 tens, in 90, 9 tens, in 120, 12 tens, in 250, 25 tens, how many any number be divided by 10?

15. How many 100's are there in 600? In 900? In 1200? In 3600? In 7500? In 8400?

16. Since in 600 there are 6 hundreds, in 900, 9 hundreds, in 1200, 12 hundreds, in 3600, 36 hundreds, how may any number be divided by 100?

104. PRINCIPLE.—*To divide by 10, 100, 1000, etc., cut off from the right of the dividend as many figures as there are ciphers on the right of the divisor.*

WRITTEN EXERCISES.

105. When the divisor is expressed by more than one figure.

1. Divide 1695 by 100.

<p>PROCESS.</p> $\begin{array}{r} 16 \overline{) 100 \, 16 \, 95} \\ \underline{160} \\ 95 \end{array}$	<p>ANALYSIS.—Since the divisor contains only hundreds, all orders lower than hundreds may be cut off from the dividend. (Prin., Art. 104.)</p> <p>Dividing, the quotient is 16, and 95 remainder, or $16\frac{95}{100}$.</p>
--	--

2. Divide 6845 by 10.

7. Divide 68459 by 100.

3. Divide 4329 by 100.

8. Divide 86937 by 100.

4. Divide 2869 by 100.

9. Divide 46973 by 1000.

5. Divide 4378 by 1000.

10. Divide 86957 by 1000.

6. Divide 5968 by 1000.

11. Divide 84549 by 1000.

12. Divide 52695 by 300.

<p>PROCESS.</p> $\begin{array}{r} 175 \overline{) 300 \, 526 \, 95} \\ \underline{175} \\ 151 \\ \underline{150} \\ 195 \end{array}$	<p>ANALYSIS.—Since the divisor contains only hundreds, all orders lower than hundreds may be cut off from the dividend. 3 hundreds are contained in 526 hundreds 175 times, and 1 hundred remainder. 1 hundred plus the partial remainder 95 is the entire remainder, 195.</p>
--	---

Hence the quotient is 175, and 195 remainder, or $175\frac{195}{300}$.

- | | |
|---------------------------|---------------------------|
| 13. Divide 6825 by 30. | 19. Divide 43869 by 4000. |
| 14. Divide 2968 by 40. | 20. Divide 38574 by 5000. |
| 15. Divide 3876 by 300. | 21. Divide 63658 by 6000. |
| 16. Divide 4986 by 400. | 22. Divide 71435 by 7000. |
| 17. Divide 89687 by 2000. | 23. Divide 88728 by 8000. |
| 18. Divide 85693 by 3000. | 24. Divide 99953 by 9000. |

25. Divide 14992 by 37.

PROCESS.

Divisor.	Dividend.	Quotient.
37)	14992	(405 $\frac{7}{37}$
	148	
	<u>192</u>	
	185	
	<u>7</u>	

ANALYSIS.—37 is not contained in 1 ten-thousand any ten-thousand times; hence we unite the ten-thousand with the thousands, making 14 thousands. 37 is not contained in 14 thousands any thousand times; hence we unite the thousands with the hundreds, making 149 hundreds.

37 is contained in 149 hundreds 4 hundred times and a remainder. The 4 is written in the quotient and the divisor multiplied by it, giving as a product 148 hundreds, which is written under units of the same order in the dividend. Subtracting, there is a remainder of 1 hundred.

The 1 hundred is united with the 9 tens, making 19 tens. 37 is not contained in 19 tens any ten times; therefore there are no tens in the quotient, and 0 is written there in tens' place.

The 19 tens are united with the 2 units, making 192 units. 37 is contained in 192 units 5 times. The 5 is written in units' place in the quotient and the divisor multiplied by it, giving for a product 185, which is written under units of the same order in the partial dividend. Subtracting, there is a remainder of 7.

Hence the quotient is 405, and 7 remainder, or $405\frac{7}{37}$.

PROOF.— $405 \times 37 + 7 = 14992$. Hence the work is correct. (Prin., Art. 100.)

106. When the steps in the solution of an example in division are written, the process is called **Long Division**.

RULE. — Write the divisor at the left of the dividend with a curved line between them.

Find how many times the divisor is contained in the fewest figures on the left hand of the dividend that will contain it, and write the quotient on the right.

Multiply the divisor by this quotient and place the product under the figures divided. Subtract the result from the partial dividend used, and to the remainder annex the next figure of the dividend.

Divide as before until all the figures of the dividend have been annexed to the remainder.

If any partial dividend will not contain the divisor, write a cipher in the quotient, then annex the next figure of the dividend and proceed as before.

If there is a remainder after the last division, write it after the quotient, or with the divisor under it as part of the quotient.

PROOF. — Multiply the divisor by the quotient, and to the product add the remainder, if any. If the work is correct, the result will equal the dividend.

1. To aid beginners in determining the quotient figure, it may be advantageous to form a table of the products of the divisor multiplied by each of the first nine numbers, as given in the margin.

The quotient figure may thus be determined readily by comparing each partial dividend with the products in the table.

$37 \times 1 =$	37.
$37 \times 2 =$	74.
$37 \times 3 =$	111.
$37 \times 4 =$	148.
$37 \times 5 =$	185.
$37 \times 6 =$	222.
$37 \times 7 =$	259.
$37 \times 8 =$	296.
$37 \times 9 =$	333.

Divide:

26. 6790 by 10.
27. 4202 by 11.
28. 2556 by 12.
29. 6480 by 15.
30. 3978 by 13.
31. 8974 by 14.
32. 1365 by 21.

Divide:

33. 2310 by 42.
34. 1488 by 31.
35. 2747 by 41.
36. 2310 by 42.
37. 3504 by 73.
38. 3796 by 52.
39. 3591 by 63.

- | | |
|--------------------|-------------------------------|
| 40. 5084 by 62. | 58. 125712 by 216. |
| 41. 3444 by 82. | 59. 203616 by 303. |
| 42. 3367 by 91. | 60. 232815 by 415. |
| 43. 3024 by 84. | 61. 205632 by 672. |
| 44. 2697 by 93. | 62. 101618 by 149. |
| 45. 56394 by 78. | 63. 238941 by 573. ✓ |
| 46. 58176 by 96. | 64. 483813 by 561. |
| 47. 57834 by 63. | 65. 347904 by 768. |
| 48. 12480 by 24. | 66. 210368 by 608. |
| 49. 50854 by 94. | 67. 218499 by 421. |
| 50. 56394 by 78. | 68. 559108 by 262. |
| 51. 59136 by 96. | 69. 1547250 by 2134. |
| 52. 90992 by 121. | 70. 2250732 by 4012. ✓ |
| 53. 13668 by 201. | 71. 3200325 by 5325. |
| 54. 13272 by 316. | 72. 1909970 by 6181. ✓ |
| 55. 39576 by 408. | 73. 2342899 by 5006. |
| 56. 209165 by 301. | 74. 1879495 by 6024. |
| 57. 475524 by 612. | 75. 3006192 by 5183. |

76. If a man paid \$7564 for 124 horses, how much did they each cost?

77. A man had 6812 pounds of flour, which he put into barrels. Since a barrel of flour is 196 pounds, how many barrels did he have?

78. A man raised 6427 bushels of oats upon 107 acres. What was the average crop per acre?

79. The combined wealth of 405 men was \$1247400. How much would each have, if they all had the same amount of money?

80. If the receipts of a railroad for 52 weeks are \$2683927, how much is that per week?

81. In a gallon of water there are 231 cubic inches. How many gallons will a cistern hold that contains 33654 cubic inches?

82. How many schooners, each carrying 8700 bushels of wheat, will be required to carry 843900 bushels?

83. If a railroad company pays \$1750 for constructing a car, how many cars can be made for \$78750?

84. The average daily receipts of a ferry-boat are \$325. In how many days will the receipts amount to \$75000?

85. The stock of a railroad company is \$2565000. If it is divided into shares of \$500 each, how many shares are there?

86. Mount Everest in Asia is said to be 29100 feet high. Since there are 5280 feet in a mile, how many miles high is it?

REVIEW.

ORAL EXERCISES.

107. 1. How many hats at \$4 apiece can be exchanged for 6 yards of broadcloth at \$6 per yard?

2. A drover bought 12 lambs for \$24 and sold them at a gain of \$12. How much did he get for each?

3. If 5 barrels of flour are worth \$40, how many yards of cloth worth \$3 per yard will 3 barrels of flour purchase?

4. A farmer exchanged 5 firkins of butter worth \$20 a firkin for some money, and flour at \$8 per barrel? How many barrels of flour did he get, if he received only \$4 in money?

5. If 6 bushels of wheat are worth \$12, how many bushels must be given for 4 tons of hay at \$8 per ton?

6. A man bought a horse for \$120. After expending \$80 in training him he sold him for \$250. How much did he gain?

7. A newsboy bought 10 papers for 35 cents and sold them so as to gain 15 cents. How much did he get apiece for them?

8. A grocer bought 7 barrels of flour at \$6 per barrel, and sold it at a gain of \$14. How much did he get per barrel for it?

9. A clothier bought 10 garments for \$50 and sold them at \$7 apiece. How much did he gain?

10. If a man earns \$12 per week, but spends \$5 for his board and \$2 for other expenses, in how many weeks will he save \$50?

11. A man sold 12 sheep at \$3 per head and 6 calves at \$4 per head, receiving his pay in five-dollar bills. How many bills did he receive?

12. I had \$40, spent \$10, and then earned enough to purchase 8 yards of cloth at \$8 per yard by using all my money. How much did I earn?

13. I purchased 10 tons of coal for \$70 and sold it so as to gain \$20. How much did I gain per ton?

14. A boat ran up a river 8 hours at the rate of 10 miles per hour and came down in 5 hours. What was its rate per hour coming down?

15. The wages of a lad are \$3 per week. In how many weeks can he earn enough to buy a suit of clothes for \$16, if he pays \$1 per week for other expenses?

16. If 10 gentlemen and 15 ladies go on an excursion, and the expenses, which are \$4 for each person, are paid by the gentlemen, how much must each gentleman pay?

17. Multiply 20 by 4, add 4, divide by 7, add 6, divide by 3, subtract 3, and add 2. What is the result?

18. Multiply 8 by 3, add 1, divide by 5, multiply by 2, add 2, divide by 3, multiply by 10. What is the result?

19. Multiply 6 by 5, add 6, divide by 9, add 6, multiply by 3, subtract 3, divide by 9. What is the result?

20. Multiply 8 by 4, subtract 2, divide by 5, add 4, divide by 2, multiply by 7, add 1, and divide by 9. What is the result?

21. Divide 36 by 9, multiply by 5, add 4, divide by 6, subtract 2, multiply by 8, subtract 4, divide by 3. What is the result?

WRITTEN EXERCISES.

108. The **Parenthesis**, (), shows that the numbers included within it are to be subjected to the same operation.

Thus, $(5-3) \times 3$ shows that $5-3$, or 2, is to be multiplied by 3.

109. The **Vinculum**, —, may be used instead of the parenthesis.

Thus $\overline{5-3} \times 3$ may be used instead of $(5-3) \times 3$.

110. Find the value of the following:

1. $(312 + 36) - (381 - 215) - 65$.
2. $(214 - 81) - (115 - 18 + 6) + 10$.
3. $(413 - 200) - (118 - \overline{24 - 4} + 6) + 3$.
4. $(171 - 86) - (\overline{3 \times 4} + 27) + 10$.
5. $(3 \times 4) \times 9 - (4 + 8) \div 3 + 5$.
6. $(5 + 2 + 6) \times 4 - (7 + 8 - 3) \div 6 + 10$.

7. If 8 horses cost \$1624, what will 5 horses cost?

PROCESS.

8 horses = \$1624
 1 horse = \$ 203
 5 horses = \$1015

EXPLANATION.—In analyzing examples like this proceed from the given number to 1 and from 1 to the required number.

8. In 15 square miles there are 9600 acres. How many acres are there in 28 square miles?

PROCESS.

15 square miles = 9600 acres
 1 square mile = 640 acres
 28 square miles = 17920 acres

9. In 52 quires of paper there are 1248 sheets. How many sheets are there in 60 quires?

PROCESS.

52 quires = 1248 sheets

1 quire = 24 sheets

60 quires = 1440 sheets

10. In 17 miles there are 29920 yards. How many yards are there in 35 miles?

11. If 20 barrels of flour weigh 3920 pounds, how much will 37 barrels weigh?

12. If 35 acres of land cost \$1750, how much will 80 acres cost at the same rate?

13. If 12 barrels of flour are worth \$132, what will 36 barrels cost?

14. If 117 tons of coal are worth \$702, how much are 235 tons worth?

15. If a farmer receives \$2275 for 13 horses, for how much would 27 horses sell at the same rate?

16. If 21 bushels of wheat weigh 1260 pounds, how much will 83 bushels weigh?

17. A man paid \$126 for 21 barrels of flour. How much would 85 barrels cost at the same rate?

18. A farmer sold 18 calves at the rate of 3 for \$33. How much did he get for them?

19. A's farm contained 227 acres. B's lacks 12 acres of being twice as much; C's lacks 42 acres of being 3 times as much as B's; and D's as much as A's and C's. How many acres were there in each farm?

20. Two men leave the same place and travel in opposite directions, one at the rate of 30 miles per day, and the other at the rate of 37 miles per day. How far will they be apart at the end of 12 days?

21. A farmer wished to obtain \$260; therefore he

72 bushels of wheat at \$1.50 per bushel and enough apples at \$2 per barrel to obtain the sum required. How many barrels of apples did he sell?

22. I bought 300 barrels of apples at \$3 per barrel. If 120 barrels decayed, at what rate must the rest be sold so that I may not lose?

23. If a book-keeper receives \$1400 per year for his services, and his expenses are \$840, in what time can he save enough to buy 32 acres at \$140 per acre?

24. Mr. B. bought 140 acres of land for \$17500, but sold enough at \$120 per acre to amount to \$9600. If he sells the rest of the land at cost, will he gain or lose, and how much?

QUESTIONS FOR REVIEW.

111. What is division? What is the dividend? What is the divisor? What is the quotient? What is the remainder? What is the sign of division? What does it show? What other ways are there of indicating division? When any thing is divided into two equal parts, what are the parts called? How are they expressed by figures? When any thing is divided into three equal parts, what is each part called? When into four equal parts, what is each part called? When in twenty-five equal parts, what is each part called? How are equal parts of a number expressed by figures?

Repeat the table of division. What is short division? What is long division? What is the principle in regard to the product of divisor and quotient, plus the remainder? What is the principle in regard to dividing by 10, 100, 1000, etc.? What is the rule for division? How is division proved?

What is the parenthesis? What does it show? What is the vinculum? How may it be used?

What is subtraction? What are the principles of subtraction? What is multiplication? What are the principles of multiplication? What is the multiplicand? What is the multiplier? What is the product?

FACTORING.

ORAL EXERCISES.

110. 1. What is the product when 3 is multiplied by 2?
What are 3 and 2 of their product?

2. What numbers multiplied together will produce 9?
What are 3 and 3 of 9?

3. What numbers multiplied together will produce 12?
What are 3 and 4, or 6 and 2, of 12?

4. What are the factors of 20? Of 36? Of 15? Of 18?

5. What are the factors of 27? Of 25? Of 32? Of 49?

6. What are the factors of 21? Of 33? Of 50? Of 35?

7. What are the factors of 63? What then is 7 of 63?

8. What is 9 of 18? What is 5 of 25? What is 8 of 32?

9. What numbers will exactly divide 18? 25? 36? 49?

10. Give the exact divisors of 42, 96, 35, 50, 27, 72.

11. Give the exact divisors of 36, 40, 48, 70, 80.

12. Give the exact divisors of 44, 56, 64, 84, 96.

13. What numbers between 0 and 10 can not be divided
by any number except themselves and 1?

14. What numbers between 10 and 20 can not be divided
by any number except themselves and 1? Between 20 and
30?

15. What numbers between 0 and 10 can be divided by
other numbers besides themselves and 1? Between 10 and
20? Between 20 and 30?

16. Select from the following the numbers which can not
be divided by any numbers beside themselves and 1: 35, 42,
37, 56, 61, 81, 93, 45, 84.

17. Select from the following the numbers that can be
divided by other numbers besides themselves and 1: 24, 36,
13, 18, 23, 37, 50, 51, 21, 85, 47, 59.

DEFINITIONS.

111. An Integer is a number that expresses whole units.

Thus, 46, 32 are integers, or integral numbers.

112. An Exact Divisor of a number is an integer that will divide it without a remainder.

Thus, 2, 3, 6, and 9 are exact divisors of 18.

113. The Factors of a number are the integers which, being multiplied together, will produce the number.

Thus, 6 and 8 are the factors of 48.

114. A Prime Number is one that has no exact divisors except itself and 1.

Thus, 1, 3, 5, 7 are prime numbers.

115. Prime Factors are factors that are prime numbers.

Thus, 3 and 5 are prime factors of 15.

116. A Composite Number is one that has exact divisors besides itself and 1.

Thus, 12 and 15 are composite numbers.

117. An Even Number is one that is exactly divisible by 2.

Thus, 8, 12, 16 are even numbers.

118. An Odd Number is one that is not exactly divisible by 2.

Thus, 1, 3, 5, 7, 9 are odd numbers.

119. Factoring is the process of separating a number into its factors.

120. An Exponent is a small figure used to indicate how many times a number is used as a factor.

Thus, 4^3 indicates that 4 is used as a factor three times.

121. PRINCIPLES.—1. *Every factor of a number is an exact divisor of it.*

2. *The only exact divisors of a number are its prime factors, or the product of two or more of them.*

WRITTEN EXERCISES.

122. 1. What are the prime factors of 336?

PROCESS.

$$2)336$$

$$2)168$$

$$2)84$$

$$2)42$$

$$3)21$$

$$7$$

ANALYSIS.—Since every factor of a number is a divisor of it (Prin. 1), we may find the prime factors of 336 by dividing by the exact divisors that are prime numbers.

Dividing, the prime factors of 336 are found to be 2, 2, 2, 3, and 7, or $2^4, 3, 7$.

RULE.—*Divide the given number by any prime number that will exactly divide it. Divide this quotient by another prime number, and so continue until the quotient is a prime number.*

The several divisors and the last quotient will be the prime factors.

What are the prime factors:

- | | | |
|-------------|--------------|--------------|
| 2. Of 168? | 12. Of 625? | 22. Of 1280? |
| 3. Of 224? | 13. Of 912? | 23. Of 1152? |
| 4. Of 144? | 14. Of 832? | 24. Of 3204? |
| 5. Of 198? | 15. Of 990? | 25. Of 2500? |
| 6. Of 216? | 16. Of 1008? | 26. Of 1296? |
| 7. Of 484? | 17. Of 1120? | 27. Of 1680? |
| 8. Of 576? | 18. Of 1176? | 28. Of 2016? |
| 9. Of 432? | 19. Of 1250? | 29. Of 2304? |
| 10. Of 672? | 20. Of 1024? | 30. Of 2436? |
| 11. Of 396? | 21. Of 1728? | 31. Of 2890? |

CANCELLATION.

ORAL EXERCISES.

123. 1. How many times is 2 times 5 contained in 4 times 5? 2 times 3 in 4 times 3? 2 times any number in 4 times that number?

2. How many times is 4 times 7 contained in 8 times 7? 4 times 35 in 8 times 35? 4 times a certain number in 8 times the same number?

3. How many times is 6×12 contained in 18×12 ? 5×23 in 15×23 ? 7 times 47 in 21 times 47?

4. What is the quotient of $(24 \times 17) \div (12 \times 17)$? Of $(63 \times 24) \div (9 \times 24)$? Of 48 times 61 divided by 24 times 61?

5. In determining the quotient, what numbers may be omitted from both dividend and divisor?

124. Cancellation is the process of shortening computations by rejecting equal factors from dividend and divisor.

125. PRINCIPLE.—*Rejecting equal factors from both dividend and divisor does not alter the quotient.*

WRITTEN EXERCISES.

126. 1. Divide $(4 \times 6 \times 8 \times 10)$ by $(2 \times 3 \times 4 \times 15)$.

PROCESS.

$$\begin{array}{ccccccc} 2 & 2 & 2 & 2 & & & \\ 4 & \times & 6 & \times & 8 & \times & 10 \\ \hline 2 & \times & 3 & \times & 4 & \times & 15 \\ & & & & & & 3 \end{array} = \frac{16}{3} = 5\frac{1}{3}.$$

ANALYSIS.—The dividend is written above the divisor, with a line between them, as in division.

Since the factors 2, 3, and 4 are factors of 4, 6, and 8 in the dividend, we reject them from both,

leaving the factors 2, 2, and 2 in the dividend. Since the factor 5 is common to 10 in the divisor and 15 in the dividend, it is also rejected, leaving 2 in the dividend and 3 in the divisor.

The product of the uncanceled factors of the dividend is 16 and the divisor 3; hence the quotient is $\frac{16}{3}$, or $16 \div 3$, or $5\frac{1}{3}$.

RULE.—*Reject from the dividend and divisor all factors common to both, and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

When all the factors of both dividend and divisor are cancelled the quotient is 1, for the dividend will then exactly contain the divisor *once*.

Divide, using cancellation :

2. $4 \times 5 \times 7 \times 9$ by $2 \times 2 \times 6 \times 7 \times 3$.
3. $8 \times 9 \times 12 \times 16$ by $4 \times 3 \times 5 \times 6 \times 20$.
4. $2 \times 3 \times 8 \times 12 \times 24$ by $6 \times 4 \times 36 \times 4$.
5. $15 \times 20 \times 25 \times 27$ by $10 \times 15 \times 18 \times 25$.
6. $18 \times 24 \times 32 \times 36$ by $9 \times 48 \times 4 \times 18$.
7. $25 \times 30 \times 40 \times 35$ by $20 \times 15 \times 18 \times 7$.
8. $40 \times 48 \times 54 \times 60$ by $30 \times 24 \times 72 \times 3$.
9. $30 \times 49 \times 64 \times 25$ by $15 \times 35 \times 24 \times 10$.
10. $12 \times 60 \times 36 \times 70$ by $28 \times 48 \times 6 \times 5$.
11. $32 \times 36 \times 33 \times 45$ by $24 \times 30 \times 44 \times 9$.
12. $40 \times 27 \times 32 \times 21$ by $24 \times 18 \times 16 \times 14$.
13. $30 \times 36 \times 24 \times 42$ by $45 \times 27 \times 8 \times 28$.
14. $27 \times 32 \times 45 \times 36$ by $18 \times 24 \times 9 \times 6$.
15. $45 \times 28 \times 36 \times 56$ by $49 \times 4 \times 18 \times 18$.
16. $27 \times 28 \times 35 \times 49$ by $18 \times 21 \times 40 \times 28$.
17. $45 \times 60 \times 63 \times 28$ by $27 \times 20 \times 25 \times 21$.
18. $55 \times 36 \times 27 \times 42$ by $12 \times 25 \times 35 \times 33$.
19. $36 \times 64 \times 25 \times 40$ by $32 \times 50 \times 18 \times 10$.
20. $56 \times 18 \times 32 \times 49$ by $16 \times 36 \times 42 \times 28$.
21. $32 \times 45 \times 72 \times 15$ by $64 \times 18 \times 36 \times 30$.
22. $64 \times 15 \times 63 \times 24$ by $30 \times 21 \times 56 \times 12$.
23. $44 \times 20 \times 50 \times 36$ by $55 \times 10 \times 25 \times 18$.
24. $56 \times 36 \times 35 \times 24$ by $40 \times 48 \times 21 \times 18$.
25. A merchant purchased 13 firkins of butter each con-

taining 39 pounds at 32 cents a pound, and paid for it with 4 patterns of silk of 13 yards each. How much was the silk worth per yard?

26. A grocer exchanged 27 chests of tea, each containing 48 pounds, worth 40 cents a pound, for 18 pieces of dress goods, each containing 60 yards. What was the price per yard of the dress goods?

27. A stationer had 28 packages of cards, each containing 24 cards, which he exchanged for 21 packages of note-paper, each containing 24 sheets, worth 2 cents per sheet. What were the cards worth apiece?

28. Four farms, each containing 80 acres, worth \$72 per acre, were exchanged for 5 farms, each containing 96 acres. What was the value per acre of the farms received in exchange?

29. A farmer exchanged 8 bushels of wheat, at \$2 per bushel, for 3 pieces of muslin, worth 10 cents per yard. How many yards were there in each piece?

QUESTIONS FOR REVIEW.

127. What is an integer? What is an exact divisor? What are factors? What is a prime number? What are prime factors? What is a composite number? What is an even number? What is an odd number? What is factoring? What is an exponent? What are the principles relating to factors? What is the rule for finding prime factors?

What is cancellation? What is the principle relating to cancellation? What is the rule for cancellation? When all the factors are cancelled, what is the quotient?

What is division? What is the dividend? What is the divisor? What is the quotient? What are the signs of division? When any thing is divided into two equal parts, what is each part called? What is each part called, when any thing is divided into seven equal parts? When into thirteen? What is the rule for long division? How is division proved?

FRACTIONS

128. 1. When an apple is divided into *two* equal parts, what is each part called?

2. How many *halves* of an apple are equal to a whole apple?

3. When a pear is divided into *three* equal parts, what is each part called? What are two of the parts called?

4. How many thirds of a pear are equal to a whole pear?

5. If an orange is cut into *four* equal parts, what is each part called? What are two of the parts called? What are three of the parts called?

6. How many fourths of an orange are equal to a whole orange?

7. When any thing is divided into *five* equal parts, what is each part called? What are three parts called?

8. When any thing is divided into *six* equal parts, what is each part called? What are five parts called?

9. How many fifths are equal to a whole?

10. When any thing is divided into *fifteen* equal parts, what is each part called? What are ten parts called?

11. How many fifteenths are equal to a whole?



12. If 8 cents are divided equally among 4 boys, what part of the money will each have?
13. How many cents are one-fifth of 10 cents?
14. How many apples are one-third of 6 apples?
15. How many boys are there in a group, if 12 boys are placed in four groups?
16. Six boys divided 12 fishes, which they had caught, equally among themselves. How many did each get? What part of the whole did each get?

DEFINITIONS.

129. A **Fraction** is one or more of the equal parts of any thing.

130. Two numbers written, one above the other, with a line between them are used to express a fraction.

131. The **Denominator** is the number which shows into how many equal parts a thing has been divided. It is written below the line.

Thus, in the fraction $\frac{3}{5}$, 5 is the denominator. It shows that something has been divided into five equal parts.

132. The **Numerator** is the number which shows how many parts form the fraction. It is written above the line.

Thus, in the fraction $\frac{3}{5}$, 3 is the numerator. It shows that the fraction contains 3 of the 5 equal parts.

133. The numerator and denominator are called the *terms of a fraction*.

134. A **Proper Fraction** is one in which the numerator is less than the denominator.

Thus, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{8}$, are proper fractions.

The value of a proper fraction is therefore less than 1.

135. An Improper Fraction is a fraction in which the numerator equals or exceeds the denominator.

Thus, $\frac{5}{3}$, $\frac{7}{4}$, $\frac{11}{8}$, are improper fractions.

The value of an improper fraction is therefore 1 or more than 1.

136. A Mixed Number is a number expressed by an integer and a fraction.

Thus, $3\frac{1}{2}$, $5\frac{3}{4}$, $6\frac{1}{3}$, are mixed numbers.

137. A fraction may be regarded as expressing *unexecuted division*.

Thus, $\frac{12}{4}$ is equal to $12 \div 4$; $\frac{18}{6}$ is equal to $18 \div 6$.

138. To read and write fractions.

1. Read the expression $\frac{5}{13}$.

EXPLANATION.—Since the denominator 13 shows that the unit has been divided into thirteen equal parts and the numerator 5, that five of the these parts form the fraction, it may be read five of the thirteen equal parts, or five thirteenths.

Read the following expressions:

$\frac{6}{7}$	$\frac{5}{9}$	$\frac{10}{13}$	$\frac{8}{23}$	$\frac{3}{18}$	$\frac{5}{24}$	$\frac{16}{27}$	$\frac{15}{30}$	$\frac{8}{26}$	$\frac{7}{42}$	$\frac{18}{29}$
$\frac{5}{4}$	$\frac{7}{11}$	$\frac{5}{8}$	$\frac{7}{9}$	$\frac{8}{20}$	$\frac{5}{13}$	$\frac{8}{21}$	$\frac{11}{20}$	$\frac{9}{22}$	$\frac{8}{36}$	$\frac{7}{24}$
$\frac{3}{10}$	$\frac{7}{14}$	$\frac{6}{20}$	$\frac{8}{39}$	$\frac{15}{32}$	$\frac{18}{40}$	$\frac{16}{17}$	$\frac{12}{23}$	$\frac{18}{25}$	$\frac{19}{42}$	$\frac{7}{21}$

Express by figures:

2. Three eighths. Seven twelfths.
3. Nine tenths. Seven elevenths.
4. Fifteen twenty-thirds. Eighteen twentieths.
5. Sixteen twenty-firsts. Fifteen forty-seconds.
6. Eighteen thirty-fifths. Nineteen fortieths.
7. Twenty-four thirty-seconds. Nine forty-fourths.
8. Twenty-six forty-thirds. Eighteen thirty-sevenths.
9. Thirty-six forty-sevenths. Twenty-four sixtieths.

10. Thirty-seven twenty-ninths. Forty-three fifty-firsts.

11. Interpret the expression $\frac{5}{7}$.

ANALYSIS.— $\frac{5}{7}$ represents 5 of the 7 equal parts, into which any thing is divided. It also represents one-seventh of 5 and 5 divided by 7. It is read *five-sevenths*.

Interpret in like manner :

12. $\frac{5}{8}$.	15. $\frac{10}{28}$.	18. $\frac{16}{37}$.	21. $\frac{36}{82}$.
13. $\frac{4}{7}$.	16. $\frac{11}{48}$.	19. $\frac{18}{43}$.	22. $\frac{14}{27}$.
14. $\frac{8}{9}$.	17. $\frac{8}{37}$.	20. $\frac{7}{10}$.	23. $\frac{18}{38}$.

REDUCTION OF FRACTIONS.

139. To reduce Fractions to their smallest or lowest terms.

1. In $\frac{1}{2}$ dollar how many fourths of a dollar are there?
In $\frac{1}{2}$ of an apple how many fourths are there?

2. In $\frac{1}{2}$ of a line how many sixths are there? How many sixths are there in $\frac{1}{2}$ of a yard?

3. How are the terms of the fraction $\frac{2}{4}$ obtained from those of $\frac{1}{2}$? $\frac{3}{6}$ from $\frac{1}{2}$?

4. Since the terms of the fraction $\frac{2}{4}$ are obtained from those of $\frac{1}{2}$, by multiplying them by 2, how may the terms of $\frac{1}{2}$ be obtained from those of $\frac{2}{4}$?

5. Since the terms of the fraction $\frac{3}{6}$ are obtained from those of $\frac{1}{2}$, by multiplying them by 3, how may the terms of $\frac{1}{2}$ be obtained from $\frac{3}{6}$?

6. Change $\frac{1}{2}$ to 8ths.

7. Change $\frac{1}{3}$ to 9ths.

8. Change $\frac{1}{4}$ to 16ths.



9. Change $\frac{4}{8}$ to halves.

10. Change $\frac{3}{6}$ to thirds.

11. Change $\frac{1}{4}$ to fourths.

140. Reduction of Fractions is the process of changing their form without changing their value.

141. A Common Divisor of two or more numbers is a number that is an exact divisor of each of them.

142. A fraction is expressed in its *Smallest* or *Lowest Terms*, when its numerator and denominator have *no common divisor*.

143. PRINCIPLE.—*Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

144. 1. Reduce $\frac{8}{80}$ to its smallest terms.

PROCESS.	ANALYSIS.
$8 \overline{) 64} = \frac{8}{8}$	to its smallest terms we divide the terms of the fraction by 8 (Prin., Art. 143) and the terms of the resulting fraction by 2. The terms of the fraction $\frac{4}{5}$ having no common divisor, $\frac{4}{5}$ when reduced to its smallest terms is equal to $\frac{4}{5}$.
$8 \overline{) 80} = \frac{10}{10}$	
$2 \overline{) 8} = \frac{4}{4}$	
$2 \overline{) 10} = \frac{5}{5}$	

RULE.—*Divide the numerator and denominator by any common divisor, and continue to divide thus, until the terms have no common divisor.*

Reduce to their smallest terms :

2. $\frac{12}{15}$.	9. $\frac{33}{42}$.	16. $\frac{480}{840}$.	23. $\frac{26}{108}$.
3. $\frac{15}{18}$.	10. $\frac{28}{42}$.	17. $\frac{180}{320}$.	24. $\frac{120}{144}$.
4. $\frac{18}{24}$.	11. $\frac{40}{72}$.	18. $\frac{144}{156}$.	25. $\frac{324}{360}$.
5. $\frac{16}{48}$.	12. $\frac{24}{44}$.	19. $\frac{240}{384}$.	26. $\frac{256}{360}$.
6. $\frac{20}{45}$.	13. $\frac{30}{55}$.	20. $\frac{630}{810}$.	27. $\frac{250}{375}$.
7. $\frac{12}{18}$.	14. $\frac{25}{45}$.	21. $\frac{125}{875}$.	28. $\frac{240}{360}$.
8. $\frac{14}{49}$.	15. $\frac{35}{60}$.	22. $\frac{450}{850}$.	29. $\frac{560}{720}$.

145. To reduce integers or mixed numbers to fractions.

1. How many halves are there in 1 apple? In 2 apples? In 4 apples?

2. How many thirds are there in 1 orange? In 2 oranges? In 4 oranges? In 6 oranges?

3. How many fourths are there in 1? In 2? In 3?

4. How many thirds are there in 1? In 3? In 5?

5. How many fifths are there in 1? In 4? In 6?

6. How many sevenths are there in 1? In $1\frac{1}{7}$? In $1\frac{2}{7}$?

7. How many eighths are there in 1? In $1\frac{1}{8}$? In $1\frac{2}{8}$?

8. How many fifths are there in 2? In $2\frac{2}{5}$? In $3\frac{1}{5}$?

9. How many sixths are there in 3? In $3\frac{2}{3}$? In $4\frac{1}{3}$?

10. How many ninths are there in 1? In $2\frac{2}{3}$? In $5\frac{1}{3}$?

WRITTEN EXERCISES.

146. 1. Reduce $17\frac{2}{5}$ to fifths.

PROCESS.

ANALYSIS.—Since in 1 there are 5 fifths, in 17 there are 17 times 5 fifths, or $\frac{85}{5}$; and in $17 + \frac{2}{5}$ there are $\frac{85}{5} + \frac{2}{5}$, or $\frac{87}{5}$.

$$17 = \frac{85}{5}$$

$$\frac{85}{5} + \frac{2}{5} = \frac{87}{5}$$

Hence $17\frac{2}{5}$ is equal to $\frac{87}{5}$.

RULE.—Multiply the integer by the given denominator, to this product add the numerator of the fractional part, and write the result over the given denominator.

Reduce the following to fractions:

2. $8\frac{5}{9}$.	7. $19\frac{3}{4}$.	12. $68\frac{10}{11}$.	17. $49\frac{15}{16}$.
3. $7\frac{6}{11}$.	8. $21\frac{2}{5}$.	13. $57\frac{13}{14}$.	18. $68\frac{13}{18}$.
4. $8\frac{9}{12}$.	9. $13\frac{3}{7}$.	14. $39\frac{21}{22}$.	19. $85\frac{13}{18}$.
5. $7\frac{8}{9}$.	10. $26\frac{5}{8}$.	15. $43\frac{16}{17}$.	20. $91\frac{22}{23}$.
6. $9\frac{5}{11}$.	11. $31\frac{4}{11}$.	16. $86\frac{21}{22}$.	21. $88\frac{8}{9}$.

147. To reduce fractions to integers or mixed numbers.

1. To how many dollars are 4 half-dollars equal?
2. To how many dollars are 6 half-dollars equal? 8 half-dollars?
3. To how many dollars are 4 quarter-dollars equal? 8 quarter-dollars? 16 quarter-dollars?
4. In 10 half-bushels, how many bushels are there? In 12 half-bushels? In 20 half-bushels?
5. How many units are there in 3 thirds? In 6 thirds? In 12 thirds? In 15 thirds?
6. How many hours are there in $\frac{1}{4}$ hours? In $\frac{1}{2}$ hours? In $\frac{2}{4}$ hours? In $\frac{3}{4}$ hours?
7. How many dollars are there in $\$ \frac{1}{4}$? In $\$ \frac{1}{2}$? In $\$ \frac{1}{4}$? In $\$ \frac{2}{4}$? In $\$ \frac{3}{4}$? In $\$ \frac{4}{4}$?
8. How many units are there in $\frac{1}{8}$? In $\frac{1}{4}$? In $\frac{1}{6}$? In $\frac{1}{8}$? In $\frac{2}{4}$? In $\frac{3}{8}$? In $\frac{4}{8}$?

WRITTEN EXERCISES.

148. 1. Reduce $2\frac{1}{9}$ to a mixed number.

PROCESS. **ANALYSIS.**—Since 9 ninths are equal to 1 unit, 218 ninths are equal to as many units as 9 ninths are contained times in 218 ninths, or $24\frac{2}{9}$ times. Therefore $2\frac{1}{9} = 24\frac{2}{9}$.

RULE.—*Divide the numerator by the denominator.*

Reduce to integers or mixed numbers:

- | | | | |
|-----------------------|------------------------|------------------------|-------------------------|
| 2. $\frac{315}{18}$. | 7. $\frac{296}{18}$. | 12. $\frac{695}{42}$. | 17. $\frac{983}{61}$. |
| 3. $\frac{216}{4}$. | 8. $\frac{345}{21}$. | 13. $\frac{869}{27}$. | 18. $\frac{697}{86}$. |
| 4. $\frac{395}{8}$. | 9. $\frac{429}{31}$. | 14. $\frac{893}{46}$. | 19. $\frac{1894}{87}$. |
| 5. $\frac{423}{11}$. | 10. $\frac{634}{27}$. | 15. $\frac{635}{71}$. | 20. $\frac{2935}{84}$. |
| 6. $\frac{617}{10}$. | 11. $\frac{889}{48}$. | 16. $\frac{845}{41}$. | 21. $\frac{6957}{98}$. |

149. To reduce dissimilar fractions to similar fractions.

1. How many fourths of an apple are there in a whole apple?

2. How many fourths are there in $\frac{1}{2}$ of an apple?

3. How many sixths are there in a whole line?



4. How many sixths are there in $\frac{1}{2}$ of a line?

5. How many sixths are there in $\frac{1}{3}$ of a line?

6. How many sixths are there in $\frac{2}{3}$ of a line?

7. How many eighths are there in a whole pie?

8. How many eighths are there in $\frac{1}{4}$ of a pie? In $\frac{3}{4}$?

9. How many twelfths are there in any thing? In $\frac{1}{2}$ of it? In $\frac{1}{3}$ of it? In $\frac{2}{3}$ of it? In $\frac{1}{4}$ of it? In $\frac{3}{4}$ of it? In $\frac{5}{6}$ of it?

10. If $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, are changed into parts of the same size, what will be the size of the parts?

11. Change $\frac{1}{2}$ and $\frac{1}{3}$ to sixths.

12. Change $\frac{1}{2}$ and $\frac{1}{4}$ to fourths.

13. Change $\frac{1}{3}$ and $\frac{1}{4}$ to twelfths.

14. Change $\frac{1}{3}$ and $\frac{1}{4}$ to fifteenths.

150. Similar Fractions are those that have the same denominator.

Thus, $\frac{2}{12}$ and $\frac{5}{12}$ are similar fractions.

151. Dissimilar Fractions are those that have not the same denominator.

Thus, $\frac{2}{3}$ and $\frac{5}{4}$ are dissimilar fractions.

152. Fractions that have the same denominator are said to have a **common denominator**.

153. When fractions have their smallest common denominator it is called their **least common denominator**.

WRITTEN EXERCISES.

154. 1. Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{12}$ to similar fractions.

PROCESS.

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}.$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}.$$

$$\frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}.$$

ANALYSIS.—Since the fractions are to be

changed to other fractions having the same denominator, we must multiply the terms of each fraction by such a number as will cause them to have the same denominator. (Prin., Art. 143.)

By examining the denominators, 4, 8, and 12, it is evident that, by multiplying 4 by 6, 8 by 3, and 12 by 2, the fractions will have a common denominator. Multiplying the terms of the fractions by these numbers severally, we have $\frac{18}{24}$, $\frac{15}{24}$, $\frac{14}{24}$.

Reduce the following to similar fractions:

2. $\frac{2}{3}$, $\frac{3}{10}$, $\frac{7}{80}$.

6. $\frac{2}{3}$, $\frac{3}{15}$, $\frac{5}{8}$.

10. $\frac{3}{11}$, $\frac{2}{4}$, $\frac{9}{22}$.

3. $\frac{5}{8}$, $\frac{1}{15}$, $\frac{7}{48}$.

7. $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{10}$.

11. $\frac{5}{12}$, $\frac{3}{8}$, $\frac{5}{6}$.

4. $\frac{5}{9}$, $\frac{2}{3}$, $\frac{7}{36}$.

8. $\frac{2}{7}$, $\frac{3}{4}$, $\frac{3}{14}$.

12. $\frac{5}{24}$, $\frac{3}{8}$, $\frac{1}{18}$.

5. $\frac{1}{10}$, $\frac{7}{20}$, $\frac{11}{80}$.

9. $\frac{3}{8}$, $\frac{5}{4}$, $\frac{5}{18}$.

13. $\frac{1}{14}$, $\frac{5}{7}$, $\frac{1}{4}$.

14. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to similar fractions having their least common denominator.

PROCESS.

$$3 = 3$$

$$4 = 2 \times 2$$

$$6 = 3 \times 2$$

$$8 = 2 \times 2 \times 2$$

$$\text{Least Com. Denom. } 2 \times 2 \times 2 \times 3 = 24.$$

ANALYSIS.—Since each

denominator is to be multiplied by some number which will produce the least common denominator, the least common denominator must be the

smallest number that will contain each of the denominators. It must therefore contain all the prime factors of the numbers, and no other factors. These factors are 3, 2×2 , 2×3 , $2 \times 2 \times 2$. Hence the smallest number which will contain all the prime factors of the given numbers, and no other factors, is $2 \times 2 \times 2 \times 3$, or 24. Hence 24 is the least common denominator.

To change the given fractions to fractions having 24 for a common denominator, the terms of the fraction $\frac{2}{3}$ must be multiplied by 8, of $\frac{3}{4}$ by 6, of $\frac{5}{6}$ by 4, of $\frac{7}{8}$ by 3. Hence the fraction becomes $\frac{16}{24}$, $\frac{18}{24}$, $\frac{20}{24}$, $\frac{21}{24}$.

RULE.—*Find the least common denominator by multiplying together the different prime factors of the given numbers. Multiply the terms of each fraction by a number which will cause each fraction to have the least common denominator.*

1. In finding the product of the different prime factors, each factor must be used the greatest number of times that it occurs in any of the given numbers.

2. Reduce all mixed numbers to improper fractions.

Reduce the following fractions to similar fractions having their least common denominator:

15. $\frac{1}{6}$, $\frac{5}{9}$, $\frac{5}{12}$. 18. $\frac{5}{28}$, $\frac{9}{42}$, $\frac{5}{21}$. 21. $1\frac{3}{8}$, $4\frac{2}{7}$, $1\frac{3}{8}$.
 16. $\frac{3}{4}$, $\frac{7}{12}$, $\frac{2}{20}$. 19. $\frac{3}{20}$, $\frac{6}{35}$, $\frac{9}{50}$. 22. $2\frac{8}{17}$, $1\frac{3}{51}$, $2\frac{1}{4}$.
 17. $\frac{4}{5}$, $\frac{7}{20}$, $\frac{11}{30}$. 20. $1\frac{3}{88}$, $\frac{11}{48}$, $\frac{17}{64}$. 23. $\frac{6}{11}$, $3\frac{3}{18}$, $1\frac{1}{2}$.

QUESTIONS FOR REVIEW.

155. What is one of the two equal parts of any thing called? One of the six equal parts? One of the nine equal parts? How many thirds make a whole? How many sixths make a whole? What is a fraction? How many numbers are required to express a fraction? How are they written? What is the denominator? Where is it written? What is the numerator? Where is it written? What are the terms of a fraction? What is a proper fraction? An improper fraction? A mixed number? How are fractional expressions interpreted?

What is reduction of fractions? What is a common divisor? When is a fraction in its smallest terms? What is the principle relating to multiplying or dividing the terms of a fraction? What is the rule for reducing a fraction to its smallest terms? What is the rule for reducing integers and mixed numbers to fractions? What is the rule for reducing fractions to mixed numbers and integers?

What are similar fractions? What are dissimilar fractions? When have fractions a common denominator? When have fractions their least common denominator? What is the rule for reducing fractions to their least common denominator? In finding the least common denominator, how many times must each prime factor of the denominators be used as a factor?

ADDITION OF FRACTIONS.

156. 1. A lad had 2-fifths of a dollar and his brother gave him 3-fifths of a dollar. How much had he then?

2. A boy worked $\frac{3}{7}$ of a day for one man and $\frac{2}{7}$ of a day for another. What part of a day did he work for both?

3. Jane solved $\frac{3}{8}$ of her problems in the afternoon and $\frac{1}{8}$ in the evening. What part of her problems had she then solved?

4. A boy spent $\frac{1}{4}$ of a dollar for a book and $\frac{1}{2}$ of a dollar for a cap. What part of a dollar did both cost?

5. A farmer sold $\frac{1}{4}$ of his flock of sheep at one time and $\frac{1}{2}$ of them at another. What part of his flock did he sell?

6. What kind of fractions can be added without changing their form?

7. What must be done to dissimilar fractions before they can be added?

157. PRINCIPLES.—1. *Only similar fractions can be added.*

2. *Dissimilar fractions must be reduced to similar fractions before adding.*

ORAL EXERCISES.

158. 1. I paid $\$ \frac{3}{4}$ for potatoes and $\$ \frac{1}{2}$ for turnips. How much did I pay for both?

2. I sold $\frac{3}{4}$ of a yard of cloth to one person and $\frac{3}{8}$ of a yard to another person. How much did I sell to both?

3. A girl earned $\$ \frac{3}{4}$ by sewing, $\$ \frac{1}{2}$ by knitting, and $\$ \frac{1}{4}$ by doing errands. How much did she earn?

4. A boy worked $\frac{2}{3}$ of a day for Mr. A. and $\frac{1}{6}$ of a day for Mr. B. How much did he work for both?

5. A man had $\frac{2}{3}$ of an acre planted with potatoes and $\frac{5}{6}$ of an acre planted with corn. How much land was planted with both?

6. A man spent $\frac{2}{3}$ of his money at one time and $\frac{1}{4}$ of it at another. What part did he spend at both times?

7. A boy caught 2 fishes, one of which weighed $\frac{3}{4}$ of a pound and the other $\frac{1}{8}$ of a pound. How much did both weigh?

8. A grocer sold $\frac{1}{2}$ dozen eggs to one man, $\frac{3}{4}$ dozen to another, and $\frac{1}{8}$ dozen to another. How many dozen did he sell them all?

9. I sold $\frac{3}{8}$ of an acre of land to one man, $\frac{1}{10}$ of an acre to another, and $\frac{7}{20}$ to another. How many acres did I sell?

10. A farmer sold $\frac{1}{3}$ of his grain to one man, $\frac{1}{5}$ of it to another, and $\frac{1}{6}$ of it to another. What part of his grain did he sell?

11. A boy paid $\frac{7}{8}$ of a dollar for a sled, $\frac{1}{4}$ of a dollar for a bow, and $\frac{1}{8}$ of a dollar for some arrows. How much did he pay for all?

12. A student paid $\$ \frac{3}{5}$ for an arithmetic, $\$ \frac{2}{3}$ for a grammar, and $\$ 1\frac{1}{2}$ for a geography. How much did he pay for all?

13. James had $\$ 3\frac{1}{2}$, Henry had $\$ 2\frac{3}{4}$, and James had $\$ 1\frac{3}{4}$. How much had they all?

14. A man saved one week $\$ 4\frac{1}{2}$, the next $\$ 3\frac{1}{2}$, and the next $\$ 5\frac{1}{2}$. How much did he save in the three weeks?

WRITTEN EXERCISES.

159. 1. What is the sum of $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{1}{5}$?

PROCESS.

$$\frac{5}{8} + \frac{3}{4} + \frac{1}{5} = \frac{25}{40} + \frac{30}{40} + \frac{8}{40} = \frac{63}{40}.$$

$$\frac{63}{40} = 2\frac{7}{40}.$$

ANALYSIS.—Since the frac-

tions are not similar, they must be changed to similar fractions before adding.

The least common denominator of the given fractions is 40, and $\frac{5}{8} = \frac{25}{40}$, $\frac{3}{4} = \frac{30}{40}$, and $\frac{1}{5} = \frac{8}{40}$. Hence the sum of the given fractions will be the sum of $\frac{25}{40}$, $\frac{30}{40}$, and $\frac{8}{40}$, which is $\frac{63}{40}$, or $2\frac{7}{40}$.

RULE.—Reduce the given fractions to similar fractions, add their numerators, and place the sum over the common denominator. When there are mixed numbers or integers, add the integers and fractions separately, and then add the results.

If the sum be an improper fraction, it should be reduced to an integer or mixed number.

Find the sum of the following:

- | | |
|---|---|
| 2. $\frac{1}{2}$, $\frac{7}{10}$, and $\frac{11}{10}$. | 10. $1\frac{3}{5}$, $2\frac{2}{7}$, $4\frac{4}{15}$, and $6\frac{2}{3}$. |
| 3. $\frac{3}{7}$, $\frac{9}{35}$, and $\frac{2}{28}$. | 11. $1\frac{5}{8}$, $3\frac{6}{7}$, $4\frac{2}{27}$, and $8\frac{2}{3}$. |
| 4. $\frac{5}{12}$, $\frac{17}{36}$, and $\frac{9}{18}$. | 12. $3\frac{3}{8}$, $5\frac{7}{8}$, $6\frac{3}{20}$, and $5\frac{1}{4}$. |
| 5. $\frac{13}{18}$, $\frac{11}{18}$, and $\frac{7}{6}$. | 13. $4\frac{1}{8}$, $6\frac{1}{4}$, $3\frac{4}{5}$, and $8\frac{7}{16}$. |
| 6. $\frac{7}{18}$, $\frac{13}{36}$, and $\frac{17}{24}$. | 14. $5\frac{1}{3}$, $6\frac{7}{36}$, $5\frac{4}{9}$, and $7\frac{2}{3}$. |
| 7. $\frac{8}{17}$, $\frac{23}{34}$, and $\frac{3}{2}$. | 15. $4\frac{1}{2}$, $5\frac{1}{5}$, $6\frac{1}{3}$, and $7\frac{1}{4}$. |
| 8. $\frac{15}{28}$, $\frac{13}{42}$, and $\frac{1}{7}$. | 16. $3\frac{7}{10}$, $5\frac{9}{50}$, $3\frac{2}{5}$, and $5\frac{13}{10}$. |
| 9. $\frac{1}{5}$, $\frac{17}{20}$, and $\frac{11}{15}$. | 17. $8\frac{1}{7}$, $3\frac{6}{7}$, $6\frac{5}{21}$, and $4\frac{2}{3}$. |

18. A farmer sold a load of hay for $\$8\frac{3}{4}$, a load of oats for $\$22\frac{3}{4}$, and a load of wheat for $\$53\frac{3}{4}$. How much did he receive for all?

19. The wages of a machinist were, for January, $\$123\frac{5}{8}$; for February, $\$96\frac{3}{4}$; for March, $\$131\frac{1}{2}$. How much did he earn in the three months?

20. An agent's traveling expenses for three weeks were as follows: the first week, $\$21\frac{2}{5}$; the second week, $\$30\frac{3}{4}$; the third week, $\$26\frac{5}{8}$. How much were his expenses for the time?

21. A man walked $26\frac{1}{3}$ miles on Monday, $13\frac{5}{7}$ miles on Tuesday, $21\frac{1}{2}$ miles on Wednesday, and $13\frac{7}{10}$ miles on Thursday. How many miles did he travel in all?

22. A man sowed $25\frac{3}{8}$ acres with wheat, $18\frac{3}{4}$ acres with oats, $19\frac{5}{8}$ acres with barley, planted $15\frac{1}{4}$ acres with corn, and raised grass upon the rest of his farm. If he had $35\frac{3}{8}$ acres of grass, how many acres were there in his farm?

SUBTRACTION OF FRACTIONS.

160. 1. Jane earned $\frac{3}{4}$ of a dollar and spent $\frac{2}{4}$ of a dollar. How much had she left?

2. A man who owned $\frac{5}{8}$ of a factory sold $\frac{3}{8}$ of it. What part did he then own?

3. If I have $\frac{3}{4}$ of a dollar and give away $\frac{1}{4}$ of a dollar, what part of a dollar will I have left?

4. A boy sold a book for $\$ \frac{1}{2}$ that cost $\$ \frac{7}{8}$. What part of a dollar did he lose?

5. A man who owned $\frac{7}{8}$ of an acre of land sold $\frac{3}{4}$ of an acre. What part of an acre had he left?

6. What kind of fractions can be subtracted without changing their form?

7. What must be done to dissimilar fractions, before they can be subtracted?

161. PRINCIPLES.—1. *Only similar fractions can be subtracted.*

2. *Dissimilar fractions must be reduced to similar fractions, before subtracting.*

ORAL EXERCISES.

162. 1. A man who had $\frac{7}{8}$ of a bushel of peaches sold $\frac{3}{8}$ of a bushel. What part of a bushel had he remaining?

2. A boy paid $\frac{3}{4}$ of a dollar for an arithmetic and $\frac{3}{4}$ of a dollar for a grammar. How much more did he pay for the one than the other?

3. If I have $\frac{9}{10}$ of a dollar and spend $\frac{1}{2}$ of a dollar, how much will I have left?

4. If I have $\frac{5}{6}$ of a dozen of marbles and lose $\frac{3}{4}$ of a dozen of them, what part of a dozen will I have left?

5. A lad hoed $\frac{5}{8}$ of a field of corn in a day. If he hoed $\frac{3}{8}$ in the forenoon, what part did he hoe in the afternoon?

6. James earns $\$5$ per day, and Henry earns $\$3$. How much more does James earn than Henry?

7. Jacob had $\$2$, but spent $\$3$. How much had he left?

8. Henry's father earned $\$3$ per day, but paid $\$7$ for his board. How much did he have left?

9. If I purchase a toy for $\$5$, giving the merchant a two-dollar bill, how much should I receive in change?

10. William had $\$10$. If he paid $\$2$ for a hat and $\$3\frac{1}{2}$ for a pair of trousers, how much did he have left?

WRITTEN EXERCISES.

163. 1. What is the difference between $\frac{9}{10}$ and $\frac{3}{8}$?

PROCESS.

$\frac{9}{10} - \frac{3}{8} = \frac{36}{40} - \frac{15}{40} = \frac{21}{40}$

ANALYSIS.—Since the fractions are not similar, they must be made similar before subtracting.

The least common denominator of the given fractions is 40, and $\frac{9}{10} = \frac{36}{40}$ and $\frac{3}{8} = \frac{15}{40}$. Hence the difference between the given fractions is the difference between $\frac{36}{40}$ and $\frac{15}{40}$, which is $\frac{21}{40}$.

2. What is the difference between $6\frac{1}{2}$ and $2\frac{5}{8}$?

PROCESS.

$6\frac{1}{2} = 6\frac{4}{8} = 5\frac{12}{8}$

$2\frac{5}{8} = 2\frac{5}{8} = 2\frac{5}{8}$

$3\frac{5}{8}$

ANALYSIS.—Since the numbers are composed of integers and fractions, each may be subtracted separately.

Reducing the fractions to similar fractions, it is evident that $\frac{1}{2}$ can not be subtracted from $\frac{5}{8}$, hence 1, taken from 6, is united

with $\frac{1}{2}$. 1 or $\frac{8}{8} + \frac{4}{8} = \frac{12}{8}$. Subtracting $2\frac{5}{8}$ from $5\frac{12}{8}$, there is a remainder of $3\frac{7}{8}$.

RULE.—Reduce the fractions to similar fractions, find the difference of the numerators, and write it over the common denominator.

When there are mixed numbers or integers, subtract the fractions and integers separately.

	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
From	$\frac{5}{8}$	$\frac{3}{7}$	$\frac{5}{8}$	$\frac{4}{5}$	$\frac{19}{80}$	$\frac{12}{11}$
Take	$\frac{3}{12}$	$\frac{2}{5}$	$\frac{3}{20}$	$\frac{7}{32}$	$\frac{8}{25}$	$\frac{6}{49}$

	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)
From	$\frac{4}{7}$	$\frac{3}{25}$	$\frac{6}{35}$	$\frac{13}{21}$	$\frac{19}{80}$	$\frac{12}{11}$
Take	$\frac{3}{49}$	$\frac{3}{70}$	$\frac{3}{28}$	$\frac{9}{56}$	$\frac{4}{52}$	$\frac{1}{80}$

15. From $6\frac{3}{4}$ take $2\frac{3}{4}$.19. From $18\frac{1}{2}$ take $9\frac{3}{4}$.16. From $9\frac{3}{4}$ take $3\frac{1}{4}$.20. From 13 take $6\frac{5}{8}$.17. From $8\frac{5}{8}$ take $2\frac{5}{24}$.21. From 22 take $8\frac{6}{13}$.18. From $16\frac{2}{3}$ take $8\frac{2}{3}$.22. From $16\frac{2}{3}$ take $10\frac{5}{6}$.

Find the value of the following:

23. $\frac{1}{4} + \frac{2}{5} - \frac{1}{3}$.

28. $3\frac{1}{4} + 2 + \frac{5}{7} - 1\frac{1}{4}$.

24. $\frac{3}{5} + \frac{2}{7} - \frac{3}{10}$.

29. $5 - 2\frac{1}{3} + 6\frac{1}{8} - \frac{5}{12}$.

25. $\frac{2}{3} + \frac{3}{5} - \frac{7}{10}$.

30. $13\frac{1}{8} - 2\frac{1}{4} - \frac{5}{8} + 2\frac{3}{8}$.

26. $\frac{5}{8} - \frac{2}{3} + \frac{3}{10}$.

31. $16\frac{1}{7} + 2\frac{1}{4} - 3\frac{5}{14} + 6\frac{1}{2}$.

27. $\frac{4}{10} - \frac{1}{3} + \frac{7}{5}$.

32. $5\frac{1}{3} - 3\frac{2}{3} + 2\frac{5}{7} - 1\frac{1}{18}$.

33. From a farm of $104\frac{5}{12}$ acres there were sold $37\frac{5}{8}$ acres. How much was left?34. A horse cost $\$147\frac{3}{4}$ and a carriage $\$189\frac{1}{2}$. How much more did the carriage cost than the horse?35. A man paid $\$219\frac{3}{4}$ for painting his house and $\$413\frac{1}{2}$ for other repairs. What was the difference between the cost of painting and other repairs?36. The net profits of a grocer for the month of January were $\$369\frac{2}{3}$; for the month of February his net profits were $\$278\frac{7}{8}$. How much more did he gain in January than in February?37. A woman having $\$25$, paid $\$2\frac{1}{2}$ for a pair of gloves, $\$3\frac{3}{4}$ for silk, and $\$8\frac{7}{8}$ for a bonnet. How much money had she left?

38. The proceeds of a sale at auction were $\$317\frac{1}{5}$, of which $\$212\frac{1}{4}$ was received in notes, $\$65$ in bank-bills, and the rest in coin. How much was received in coin?

39. A clerk earned $\$60$ per month. His expenses during that time were as follows: board $\$20\frac{3}{8}$, washing $\$5\frac{3}{4}$, other expenses $\$5\frac{1}{2}$. How much did he save per month?

40. The expenses of an office were as follows: for rent $\$18\frac{3}{5}$, for cleaning $\$3\frac{1}{4}$, for clerk hire $\$62\frac{7}{8}$, for stationery $\$2\frac{7}{10}$. If the income was $\$100$, how much were the net proceeds?

MULTIPLICATION OF FRACTIONS.

164. To multiply a fraction by an integer.

1. At 2 fifths of a dollar per yard, what will 2 yards of cloth cost?

2. At $\frac{2}{3}$ of a dollar per pound, what will 2 pounds of tea cost?

3. If a man can earn $\frac{3}{10}$ of a dollar per hour, how much can he earn in 2 hours? How much is 2 times $\frac{3}{10}$? 3 times $\frac{3}{10}$?

4. How much is 5 times $\frac{2}{18}$? 6 times $\frac{2}{18}$? 7 times $\frac{2}{18}$?

5. How, then, may a fraction be multiplied by any number?

6. How much is 4 times $\frac{3}{8}$? Express the result in smallest terms. How may this result be obtained from $\frac{3}{8}$?

7. How much is 5 times $\frac{2}{5}$? Express the result in smallest terms. How may this result be obtained from $\frac{2}{5}$?

8. In what other way, besides by multiplying the numerator, may a fraction be multiplied?

165. PRINCIPLE.—*Multiplying the numerator or dividing the denominator of a fraction by any number, multiplies the fraction by that number.*

ORAL EXERCISES.

166. 1. If a man spends $\$ \frac{3}{4}$ per day, how much will he spend in 8 days?

2. At $\$ \frac{5}{8}$ each, how much will 7 baskets cost?

3. If paper is worth $\$ \frac{2}{3}$ per package, how much will 10 packages cost?

4. What will 9 yards of cloth cost at $\$ 1 \frac{1}{2}$ per yard?

5. If a man can mow $\frac{2}{3}$ of an acre of grass in a day, how many acres can 12 men mow in the same time?

6. If molasses is selling at $\$ \frac{4}{5}$ per gallon, how much must be paid for 10 gallons?

7. If a man pays $\$ \frac{9}{10}$ for a book, how much will 20 such books cost?

8. When barley is worth $\$ 1 \frac{1}{2}$ per bushel, how much will 24 bushels be worth?

9. If a man earns $\$ 1 \frac{1}{2}$ per day, how much will he earn in 6 days?

10. If sugar is worth $8 \frac{1}{2}$ cents per pound, how much are 5 pounds worth?

11. When oranges are selling at $12 \frac{1}{2}$ cents per dozen, how much will 5 dozen cost?

12. At $\$ 6 \frac{2}{3}$ per barrel, how much will 8 barrels of flour cost?

13. What will 9 dozen of eggs cost at $10 \frac{1}{2}$ cents per dozen?

14. What will 8 tons of hay cost at $\$ 8 \frac{5}{8}$ per ton?

15. When hats sell at $\$ 3 \frac{1}{5}$ apiece, how much will a hatter receive for 7 hats?

16. If a man walks $3 \frac{2}{5}$ miles per hour, how far will he walk in 6 hours?

17. If shingles are worth $\$ 4 \frac{2}{3}$ per thousand, how much will 8 thousand cost?

18. What will $3 \frac{1}{3}$ pounds of rice cost at 9 cents per pound?

WRITTEN EXERCISES.

167. 1. Multiply $\frac{16}{21}$ by 7.

PROCESS.

$$\frac{16}{21} \times 7 = \frac{16 \times 7}{21} = \frac{112}{21} = 5\frac{1}{3}$$

Or,

$$\frac{16}{21} \times 7 = \frac{16}{21 \div 7} = \frac{16}{3} = 5\frac{1}{3}$$

ANALYSIS.—7 times 16 twenty-firsts are 112 twenty-firsts, or $5\frac{1}{3}$.

Or,

Since dividing the denominator multiplies the fraction (Prin.), 7 times $\frac{16}{21}$ are $\frac{16}{3}$, or $5\frac{1}{3}$.

RULE.—*Multiply the numerator or divide the denominator of the fraction by the integer.*

1. In multiplying mixed numbers, multiply the integers and fractions separately, and add the results.

2. When possible, use cancellation.

Multiply:

2. $\frac{9}{25}$ by 5.

3. $\frac{5}{27}$ by 4.

4. $\frac{14}{33}$ by 5.

5. $\frac{11}{24}$ by 8.

Multiply:

6. $\frac{15}{8}$ by 7.

7. $\frac{17}{9}$ by 6.

8. $\frac{21}{10}$ by 10.

9. $\frac{15}{8}$ by 9.

Multiply:

10. $2\frac{3}{4}$ by 8.

11. $6\frac{5}{6}$ by 3.

12. $18\frac{3}{4}$ by 6.

13. $22\frac{5}{8}$ by 9.

14. What is the value of 18 yards of cloth at $\$1\frac{1}{2}$ per yard?15. What will 15 magazines cost at $\$1\frac{5}{6}$ each?16. How much will a man earn in 12 days, if he gets $\$2\frac{1}{2}$ per day?17. If a railway train moves at the rate of $20\frac{1}{2}$ miles per hour, how far will it go in 10 hours?18. How much will 18 yards of silk cost at $\$4\frac{1}{2}$ per yard?19. If a man works $8\frac{3}{4}$ hours per day, how many hours will he work in 14 days?20. How much must be paid for board for 12 weeks at $\$7\frac{1}{2}$ per week?21. How much will 13 tons of anthracite coal cost at $\$6\frac{1}{2}$ per ton?

22. How much will a man earn in 12 months, if his wages are $\$68\frac{3}{4}$ per month?

23. When flour is $\$8\frac{3}{4}$ per barrel, how much must be paid for 15 barrels?

24. If the railroad fare between two places is $\$6\frac{3}{10}$, what will the fare of 9 persons cost?

168. To multiply an integer by a fraction.

ORAL EXERCISES.

1. William had 9 pencils, but lost $\frac{1}{3}$ of them. How many did he lose?

2. James had 12 chickens, but sold $\frac{1}{4}$ of them. How many did he sell?

3. A bookseller, who had 12 books of the same kind, sold $\frac{1}{3}$ of them. How many did he sell? If he had sold $\frac{2}{3}$ of them, how many would he have sold?

4. How much will $\frac{1}{4}$ of a pound of sugar cost at 12 cents a pound? How much will $\frac{3}{4}$ of a pound cost?

5. Henry had 15 cents, but he spent $\frac{1}{5}$ of the amount for a pencil. How much did he spend? If he had spent $\frac{2}{5}$ of what he had, how much would he have spent?

6. Mary had 8 chickens, but $\frac{3}{4}$ of them died. How many died?

7. A man earned \$15 per week and paid $\frac{2}{5}$ of the amount for his board. How much did his board cost him?

8. A man paid \$16 for a coat and $\frac{5}{8}$ as much for other garments. What did the other garments cost him?

9. The expenses of a dry-goods merchant were \$18 per day and the expenses of a grocer $\frac{7}{8}$ as much. What were the grocer's expenses?

10. David earned during the summer \$20 and spent $\frac{2}{5}$ of the sum for clothing. How much did his clothing cost?

11. A servant worked for 24 days and was then ill $\frac{7}{11}$ as many days as she worked. How many days was she ill?

12. Oscar solved 22 examples, but James solved only $\frac{7}{11}$ as many. How many did James solve?

13. David lost 30 marbles, but afterward found $\frac{5}{6}$ of them. How many did he find?

14. Newton paid 20 cents for a slate and $\frac{7}{8}$ as much for a book. How much did he pay for the book?

15. A man paid \$6 for coal and $2\frac{1}{2}$ times as much for provisions. How much did he pay for provisions?

16. A lady paid \$10 for a bonnet and $3\frac{1}{2}$ times as much for a dress. How much did her dress cost her?

WRITTEN EXERCISES.

169. 1. Multiply 36 by $\frac{5}{9}$, or find $\frac{5}{9}$ of 36.

$$36 \times \frac{5}{9} = \frac{\overset{4}{\cancel{36}} \times 5}{\cancel{9}} = 20$$

PROCESS.

ANALYSIS.—To multiply 36 by $\frac{5}{9}$ is to find $\frac{5}{9}$ of 36. $\frac{1}{9}$ of 36 is 4, and $\frac{5}{9} = 5$ times 4, or 20. Or,

Since $\frac{5}{9}$ is $\frac{1}{3}$ of 5, $\frac{5}{9}$ of 36 = $\frac{1}{3}$ of 5 times 36, or $\frac{1}{3} \times 180 = 60$.

RULE.—*Multiply the integer by the numerator of the fraction and divide the product by the denominator.*

1. When the multiplier is a mixed number, multiply by the integer and the fraction separately, and add the results.

2. When possible, use cancellation.

Multiply:

2. 12 by $\frac{5}{8}$.

3. 24 by $\frac{7}{8}$.

4. 36 by $\frac{5}{12}$.

5. 19 by $\frac{7}{8}$.

6. 105 by $\frac{6}{15}$.

7. 288 by $\frac{1}{2}$.

Multiply:

8. 56 by $\frac{1}{2}$.

9. 64 by $\frac{1}{8}$.

10. 76 by $\frac{1}{10}$.

11. 81 by $\frac{1}{9}$.

12. 60 by $\frac{2}{5}$.

13. 72 by $\frac{1}{3}$.

Multiply:

14. 12 by $5\frac{1}{2}$.

15. 16 by $6\frac{1}{8}$.

16. 18 by $6\frac{1}{2}$.

17. 24 by $7\frac{1}{2}$.

18. 29 by $8\frac{1}{2}$.

19. 61 by $9\frac{1}{2}$.

20. A mill was valued at \$6570. How much was the value of $\frac{2}{3}$ of it?

21. The expenses of a pleasure excursion were \$364. What would $\frac{3}{4}$ of the expenses be?

22. A drover bought $6\frac{2}{3}$ car-loads of cattle, paying on an average \$1024 per car. How much did he pay for all?

23. The average rate per hour of a fast express train was 45 miles. How far would it go in $6\frac{3}{4}$ hours?

24. If a mowing machine costs \$60 and a reaper $3\frac{1}{4}$ times as much, how much does a reaper cost?

25. What will be the cost of $9\frac{1}{2}$ yards of silk at \$3 per yard, $4\frac{1}{2}$ yards of broadcloth at \$7 per yard, and $13\frac{1}{4}$ yards of cassimere at \$2 per yard?

26. What will be the cost of $6\frac{1}{2}$ dozen arithmetics at \$6 per dozen, $5\frac{3}{4}$ dozen grammars at \$8 per dozen, and $6\frac{1}{2}$ dozen geographies at \$9 per dozen?

170. To multiply a fraction by a fraction.

ORAL EXERCISES.

1. If $\frac{1}{2}$ yard is divided into 2 equal parts, what part of a yard will each part be? How much is $\frac{1}{2}$ of $\frac{1}{2}$ of a yard?

2. If $\frac{1}{4}$ of a yard is divided into 2 equal parts, what part of a yard will each part be? How much is $\frac{1}{2}$ of $\frac{1}{4}$?

3. If $\frac{1}{3}$ of an apple is divided into 2 equal parts, what part of the apple will each part be? How much is $\frac{1}{2}$ of $\frac{1}{3}$?

4. If $\frac{1}{2}$ of a cake is divided into 3 equal parts, what part of the cake will each part be? How much is $\frac{1}{3}$ of $\frac{1}{2}$?

5. How much is $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{3}$?

6. How much is $\frac{1}{3}$ of $\frac{1}{2}$? Since $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$, how much is $\frac{2}{3}$ of $\frac{1}{2}$? $\frac{3}{3}$ of $\frac{1}{2}$? $\frac{4}{3}$ of $\frac{1}{2}$?

7. A man having $\frac{1}{2}$ acre of land sold $\frac{2}{3}$ of it. What part of an acre did he sell?

8. If a yard of crape costs $\$ \frac{1}{2}$, what part of a dollar will $\frac{2}{3}$ of a yard cost?

9. From a cistern $\frac{1}{2}$ full of water $\frac{2}{3}$ of the quantity leaked out. What part of the cistern was emptied?

10. How much is $\frac{1}{3}$ of $\frac{1}{4}$? Since $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$, how much is $\frac{1}{3}$ of $\frac{2}{3}$? $\frac{1}{3}$ of $\frac{4}{5}$? $\frac{1}{3}$ of $\frac{5}{6}$? $\frac{2}{3}$ of $\frac{2}{3}$? $\frac{2}{3}$ of $\frac{5}{6}$?

11. When cloth is worth $\$ \frac{7}{8}$ per yard, what will $\frac{1}{4}$ of a yard cost? What will $\frac{3}{4}$ of a yard cost?

12. Mr. Stone having a lot containing $\frac{3}{4}$ of an acre sold $\frac{1}{2}$ of it. What part of an acre did he sell? If he had sold $\frac{2}{3}$ of it, what part of an acre would he have sold?

13. If a boy who had $\$ \frac{4}{5}$ spent $\frac{1}{3}$ of it for candy, what part of a dollar did he spend? If he had spent $\frac{2}{3}$ of it, what part of a dollar would he have spent?

14. Mr. Stearns, who owned $\frac{2}{3}$ of a store, sold $\frac{2}{3}$ of his share. What part of the whole store did he sell?

15. What part of his farm does a man sell who sells $\frac{2}{3}$ of $\frac{3}{4}$ of it?

16. What part of a dollar is $\frac{2}{3}$ of $\$ \frac{3}{4}$?

WRITTEN EXERCISES.

171. 1. Multiply $\frac{5}{8}$ by $\frac{4}{7}$, or find $\frac{4}{7}$ of $\frac{5}{8}$.

PROCESS.

$$\frac{5}{8} \times \frac{4}{7} = \frac{5 \times 4}{8 \times 7} = \frac{5}{14}$$

ANALYSIS.—To multiply $\frac{5}{8}$ by $\frac{4}{7}$ is to find $\frac{4}{7}$ of $\frac{5}{8}$, or 4 times $\frac{1}{7}$ of $\frac{5}{8}$. $\frac{1}{7}$ of $\frac{5}{8} = \frac{5}{8 \times 7}$ and $\frac{4}{7}$ of $\frac{5}{8} = \frac{4 \times 5}{8 \times 7} = \frac{5}{14}$.

RULE.—Reduce all integers and mixed numbers to improper fractions.

Multiply the numerators together for the numerator of the product and the denominators together for its denominator.

When possible use cancellation.

172. Integers may be expressed as fractions by writing 1 as a denominator.

Thus, 4 may be written as $\frac{4}{1}$.

173. The word *of* between the fractions is equivalent to the *sign of multiplication*. Such expressions are sometimes called *Compound Fractions*.

Thus $\frac{3}{4}$ of $\frac{5}{7}$ is equal to $\frac{3}{4} \times \frac{5}{7}$.

Find:

2. $\frac{3}{4}$ of $\frac{4}{5}$.

3. $\frac{5}{7}$ of $\frac{7}{8}$.

4. $\frac{7}{9}$ of $\frac{14}{15}$.

5. $\frac{8}{9}$ of $\frac{8}{7}$.

Find:

6. $\frac{1}{7}$ of $\frac{5}{8}$.

7. $\frac{1}{4}$ of $\frac{8}{5}$.

8. $\frac{1}{3}$ of $\frac{4}{7}$.

9. $\frac{1}{15}$ of $\frac{35}{4}$.

Find:

10. $\frac{3}{8}$ of $\frac{3}{4}$ of $\frac{4}{5}$.

11. $\frac{3}{8}$ of $\frac{5}{7}$ of $\frac{7}{8}$.

12. $\frac{5}{8}$ of $\frac{7}{9}$ of $\frac{3}{5}$.

13. $\frac{8}{9}$ of $\frac{5}{16}$ of $\frac{8}{15}$.

Multiply:

14. $\frac{5}{18}$ by $\frac{1}{7}$.

15. $\frac{7}{28}$ by $\frac{9}{85}$.

16. $\frac{16}{19}$ by $\frac{1}{21}$.

Find the value of:

17. $\frac{24}{35} \times \frac{7}{15} \times \frac{15}{8} \times \frac{3}{20} \times \frac{5}{7}$.

18. $\frac{1}{25} \times \frac{5}{7} \times \frac{8}{9} \times \frac{3}{11} \times \frac{5}{8}$.

19. $\frac{45}{11} \times \frac{1}{16} \times \frac{2}{25} \times \frac{1}{11} \times \frac{1}{15}$.

20. Multiply $\frac{5}{7}$ of $\frac{3}{8}$ of 4 by $\frac{2}{9}$ of $\frac{3}{4}$ of $5\frac{1}{2}$.

21. Multiply $\frac{8}{9}$ of $\frac{4}{5}$ of $3\frac{1}{4}$ by $\frac{1}{15}$ of $\frac{1}{7}$ of $7\frac{1}{2}$.

22. Multiply $\frac{5}{8}$ of $\frac{3}{10}$ of 16 by $\frac{8}{15}$ of $\frac{1}{2}$ of $4\frac{1}{2}$.

23. Multiply $\frac{3}{11}$ of $\frac{4}{5}$ of $2\frac{1}{2}$ by $\frac{5}{9}$ of $\frac{8}{15}$ of 6.

24. A man having $387\frac{1}{2}$ acres of land sold $\frac{3}{8}$ of it. How many acres did he sell?

25. When hay is worth \$12 $\frac{3}{4}$ per ton, how much will $\frac{7}{8}$ of a ton cost?

26. There are $16\frac{1}{2}$ feet in a rod. How many feet are there in $3\frac{1}{2}$ rods?

27. A man purchased $\frac{3}{8}$ of 420 $\frac{3}{4}$ acres of land and then sold $\frac{3}{8}$ of what he had bought. How many acres did he sell?

28. At an auction sale a cow was sold for \$39 $\frac{3}{4}$ and a horse for $3\frac{3}{4}$ times as much. For how much was the horse sold?

29. If a man has paid \$31 $\frac{5}{8}$, but owes 5 $\frac{3}{8}$ times as much as he has paid, how much does he owe?

30. What will 32 $\frac{5}{8}$ yards of imported broadcloth cost at \$4 $\frac{1}{2}$ per yard?

31. A grain merchant purchased 316 $\frac{5}{8}$ bushels of barley in one week and 5 $\frac{3}{8}$ times as much wheat. How much wheat did he purchase?

DIVISION OF FRACTIONS.

174. To divide a fraction by an integer.

1. A man divided 3 fourths of a dollar equally among 3 boys. What part of a dollar did each receive? How much is $\frac{3}{4} \div 3$?

2. A man divided $\frac{3}{8}$ of an acre into 3 equal lots. How large was each?

3. Henry divided $\frac{3}{4}$ of a melon equally among 3 boys. What part of a melon did each receive? How much is $\frac{3}{4} \div 3$?

4. In dividing a fraction, what part of the fraction is divided?

5. If $\frac{1}{2}$ of an orange is divided equally between two boys, what part of an orange does each receive?

6. If $\frac{1}{2}$ of an estate is divided equally among 4 children, what part of the estate does each receive? How much is $\frac{1}{2} \div 4$?

7. If $\frac{1}{2}$ of a farm is divided into 5 equal fields, what part of the farm is in each field? How much is $\frac{1}{2} \div 5$?

8. In what other way, besides by dividing the numerator can a fraction be divided?

175. PRINCIPLE. -- *Dividing the numerator or multiplying the denominator of a fraction by any number divides the fraction by that number.*

ORAL EXERCISES.

176. 1. If 4 pounds of tea cost $\$ \frac{8}{9}$, what will 1 pound cost?
 2. If 3 dozen oranges can be bought for $\$ \frac{9}{7}$, what do they cost per dozen?
 3. If 5 yards of muslin cost $\$ \frac{4}{5}$, what will 1 yard cost?
 4. James sold 8 quarts of berries for $\$ \frac{4}{5}$. How much did he receive per quart for them?
 5. If $\frac{7}{8}$ of a barrel of flour is divided equally among 3 persons, what part of a barrel will each receive?
 6. When $\$ \frac{8}{9}$ is paid for 4 pounds of coffee, what is the price per pound?
 7. If 5 slates can be bought for $\$ 1 \frac{9}{10}$, what is the price of each?
 8. When 5 pounds of sugar can be bought for $\$ 1 \frac{7}{10}$, what is the price per pound?
 9. If 3 yards of linen cost $\$ \frac{9}{4}$, what is the price per yard?
 10. If 3 bushels of potatoes are sold for $\$ \frac{5}{7}$, what is the price per bushel?
 11. If 5 bushels of oats are purchased for $\$ \frac{9}{7}$, what is the price per bushel?

WRITTEN EXERCISES.

177. 1. Divide $\frac{14}{15}$ by 7, or find $\frac{1}{7}$ of $\frac{14}{15}$.

PROCESS.

$$\frac{14}{15} \div 7 = \frac{14 \div 7}{15} = \frac{2}{15}$$

Or,

$$\frac{14}{15} \div 7 = \frac{14}{15 \times 7} = \frac{2}{15}$$

ANALYSIS.—Since dividing the numerator of a fraction divides the fraction (Prin.), the fraction $\frac{14}{15}$ may be divided by 7, by dividing the numerator by 7. Hence the result is $\frac{2}{15}$. Or,

Since multiplying the denominator divides the fraction, the fraction may be divided by 7, by multiplying the denominator by 7. The result by both processes is the same.

RULE.—*Divide the numerator or multiply the denominator of the fraction by the given integer.*

Divide:

2. $1\frac{1}{7}$ by 4.

3. $1\frac{1}{3}$ by 6.

4. $1\frac{1}{2}$ by 5.

5. $4\frac{1}{7}$ by 8.

Divide:

6. $3\frac{1}{11}$ by 13.

7. $1\frac{1}{10}$ by 15.

8. $8\frac{1}{3}$ by 17.

9. $7\frac{1}{9}$ by 19.

Divide:

10. $1\frac{1}{2}$ by 21.

11. $3\frac{1}{7}$ by 18.

12. $2\frac{1}{4}$ by 36.

13. $1\frac{1}{3}$ by 29.

14. Divide $11\frac{3}{4}$ by 4.

PROCESS.

$$11\frac{3}{4} = \frac{47}{4}$$

$$\frac{47}{4} \div 4 = \frac{47}{16} = 2\frac{15}{16}$$

Or,

$$\begin{array}{r} 4 \overline{) 11\frac{3}{4}} \\ \underline{2\frac{15}{16}} \end{array}$$

ANALYSIS.—The mixed number may be reduced to an improper fraction and the process continued as before. Or,

The number may be divided without being reduced to an improper fraction. Thus, 4 is contained in 11, 2 times and a remainder of $3\frac{1}{4}$, or $\frac{13}{4}$, and $\frac{13}{4}$ divided by 4 equals $\frac{15}{16}$.

Divide:

15. $21\frac{1}{7}$ by 6.

16. $13\frac{1}{2}$ by 4.

17. $19\frac{1}{7}$ by 5.

Divide:

18. $36\frac{2}{3}$ by 9.

19. $29\frac{5}{11}$ by 8.

20. $61\frac{8}{9}$ by 7.

Divide:

21. $264\frac{1}{3}$ by 6.

22. $182\frac{1}{3}$ by 8.

23. $269\frac{1}{7}$ by 5.

24. A laborer earned $\$15\frac{1}{4}$ in 9 days. How much did he earn per day?

25. Eight dozen books cost $\$90\frac{1}{2}$. What was the cost per dozen?

26. A farmer sold 12 bushels of wheat for $\$19\frac{1}{2}$. How much did he receive for it per bushel?

27. A grocer paid $\$8\frac{1}{2}$ for 20 pounds of butter. How much did he pay per pound?

28. The profits of a business for a month were $\$623\frac{1}{2}$. If the sum is divided equally among 4 partners, what is the share of each?

29. A father divided equally among his 4 children $\$367\frac{1}{2}$. How much did each receive?

178. To divide an integer by a fraction.**ORAL EXERCISES.**

1. How many eighths are there in a dollar? At $\$ \frac{1}{8}$ each, how many books can be bought for \$1? For \$2?
2. If butter is worth $\$ \frac{1}{4}$ per pound, how many pounds can be bought for \$1? For \$2? For \$3?
3. If a man can mow $\frac{1}{4}$ of a field in one day, how long will it take him to mow the whole field?
4. At $\$ \frac{1}{6}$ each, how many slates can be bought for \$1? How many at $\$ \frac{2}{3}$? How many at $\$ \frac{3}{4}$?
5. At $\$ \frac{1}{3}$ per hour, in how many hours can a man earn \$1? At $\$ \frac{2}{3}$?
6. At $\$ \frac{1}{4}$ per yard, how many yards of ribbon can be bought for \$1? At $\$ \frac{2}{7}$? At $\$ \frac{3}{7}$? At $\$ \frac{4}{7}$? At $\$ \frac{5}{7}$?
7. Six bushels of potatoes were distributed among some poor people, each person receiving $\frac{2}{3}$ of a bushel. How many persons were there?
8. At $\$ \frac{2}{3}$ per gallon, how many gallons of molasses can be bought for \$10?
9. When rye is worth $\$ \frac{7}{8}$ per bushel, how many bushels can be bought for \$4?
10. How many times is $\frac{3}{4}$ contained in 6? In 8? In 10?

WRITTEN EXERCISES.

179. 1. Divide 10 by $\frac{5}{6}$, or find how many times $\frac{5}{6}$ is contained in 10.

PROCESS.

$$10 \div \frac{5}{6} = \frac{10 \times 6}{5} = 12$$

Or,

$$10 = \frac{60}{6}; \frac{60}{6} \div \frac{5}{6} = 12$$

ANALYSIS.—Since $\frac{1}{6}$ is contained in 1, 6 times, it is contained in 10, 10 times 6 times, or 60 times, and $\frac{5}{6}$ is contained in 10, $\frac{1}{5}$ of 60 times, or 12 times. Or,

Since in 10 there are $\frac{60}{6}$, $\frac{5}{6}$ is contained in $\frac{60}{6}$, 12 times.

RULE.—*Multiply the integer by the denominator of the fraction and divide the product by the numerator. Or,*

Reduce the dividend and divisor to similar fractions and divide the numerator of the dividend by the numerator of the divisor.

1. *When possible use cancellation.*

2. Reduce mixed numbers to improper fractions.

Divide:

Divide:

Divide:

2. 12 by $\frac{3}{8}$.

7. 13 by $\frac{3}{8}$.

12. 26 by $\frac{4}{11}$.

3. 18 by $\frac{3}{8}$.

8. 23 by $\frac{7}{8}$.

13. 18 by $\frac{8}{15}$.

4. 21 by $\frac{3}{7}$.

9. 52 by $\frac{4}{5}$.

14. 34 by $\frac{10}{11}$.

5. 36 by $\frac{9}{16}$.

10. 36 by $\frac{5}{9}$.

15. 56 by $\frac{12}{13}$.

6. 45 by $\frac{5}{8}$.

11. 72 by $\frac{1}{16}$.

16. 39 by $\frac{2}{15}$.

17. How many times is $3\frac{1}{2}$ contained in 28?

18. How many times is $4\frac{1}{3}$ contained in 30?

19. How many times is $3\frac{1}{5}$ contained in 24?

20. How many times is $4\frac{1}{6}$ contained in 20?

21. How many times is $8\frac{1}{3}$ contained in 100?

22. How many lots containing $\frac{3}{4}$ of an acre each can be formed out of a lot containing 12 acres?

23. How many peaches at $\frac{5}{8}$ of a cent each can be bought for 20 cents?

24. When hay is worth $\$10\frac{1}{2}$ per ton, how many tons can be bought for $\$45$?

25. How many cords of wood at $\$3\frac{3}{4}$ per cord can be bought for $\$29$?

26. How many pairs of boots at $\$3\frac{2}{5}$ a pair can be bought for $\$34$?

27. How many pictures worth $\$4\frac{2}{3}$ apiece can be purchased for $\$56$?

28. When apples are worth $\$3\frac{2}{5}$ per barrel, how many barrels must a farmer sell so that he may obtain $\$54$ with which to pay his taxes?

180. To divide a fraction by a fraction.

ORAL EXERCISES.

1. How many pieces of ribbon $\frac{1}{3}$ of a yard long can be made from a piece $\frac{2}{3}$ of a yard long?

2. How many times can a cup, holding $\frac{1}{8}$ of a gallon, be filled from a jar holding $\frac{7}{8}$ of a gallon of vinegar?

3. How many times can a boy fill a bag, which holds $\frac{1}{4}$ of a quart, from a box that contains $\frac{4}{5}$ of a quart of cherries?

4. How many times is 1 fifth contained in 7 fifths? $\frac{2}{3}$ in $\frac{8}{9}$? $\frac{3}{5}$ in $\frac{2}{5}$? $\frac{5}{7}$ in $\frac{10}{7}$? $\frac{7}{12}$ in $\frac{21}{12}$? $\frac{9}{18}$ in $\frac{12}{18}$?

5. How many times is $\frac{1}{8}$ contained in 1?

6. Since $\frac{1}{8}$ is contained in 1, eight times, how many times will it be contained in $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{1}{3}$? In $\frac{1}{6}$?

7. How many times is $\frac{3}{8}$ contained in 1?

8. Since $\frac{3}{8}$ is contained in 1, $\frac{1}{3}$ of 8 times, or $\frac{8}{3}$ times, how many times will it be contained in $\frac{1}{2}$? In $\frac{1}{3}$? In $\frac{1}{4}$? In $\frac{2}{3}$?

9. How many times is $\frac{1}{5}$ contained in 1? $\frac{2}{5}$ in 1? $\frac{3}{5}$ in 1?

10. How many times is $\frac{1}{5}$ contained in $\frac{1}{2}$? $\frac{2}{5}$ in $\frac{1}{2}$? $\frac{3}{5}$ in $\frac{1}{2}$?

11. How many times is $\frac{1}{5}$ contained in $\frac{1}{3}$? $\frac{2}{5}$ in $\frac{1}{3}$? $\frac{3}{5}$ in $\frac{1}{3}$?

12. At $\$ \frac{3}{10}$ per pound, how many pounds of tea can be bought for $\$ \frac{7}{5}$?

13. How many quires of paper at $\$ \frac{1}{4}$ per quire can be bought for $\$ \frac{3}{2}$?

14. How many pounds of honey at $\$ \frac{3}{16}$ per pound can be bought for $\$ \frac{7}{8}$?

15. At $\$ \frac{2}{3}$ per yard, how many yards of cloth can be bought for $\$ \frac{9}{10}$?

16. How many times is $\frac{3}{4}$ contained in $\frac{11}{4}$?

WRITTEN EXERCISES.

181. 1. Divide $\frac{3}{8}$ by $\frac{2}{3}$, or find how many times $\frac{2}{3}$ is contained in $\frac{3}{8}$?

PROCESS.

$$\frac{3}{8} \div \frac{2}{3} = \frac{3}{8} \times \frac{3}{2} = \frac{9}{16}$$

Or,

$$\frac{3}{8} \div \frac{2}{3} = \frac{9}{16} \div \frac{10}{16} = \frac{9}{10}$$

ANALYSIS.—Since $\frac{1}{3}$ is contained in 1 3 times, $\frac{2}{3}$ is contained in 1, one-half of 3 times, or $\frac{3}{2}$ times.

Since $\frac{2}{3}$ is contained in 1 $\frac{3}{2}$ times, in $\frac{3}{8}$ it will be contained $\frac{3}{8}$ of $\frac{3}{2}$ times, or $\frac{9}{16}$ times. Or,

$\frac{3}{8}$ is equal to $\frac{9}{16}$ and $\frac{2}{3}$ is equal to $\frac{10}{16}$. $\frac{9}{16}$ divided by $\frac{10}{16}$ is equal to $\frac{9}{10}$.

RULE.—Multiply the dividend by the divisor inverted. Or,

Reduce the dividend and divisor to similar fractions and divide the numerator of the dividend by the numerator of the divisor.

1. When possible, use cancellation.

2. Reduce mixed numbers and integers to improper fractions.

Divide:

2. $\frac{3}{8}$ by $\frac{5}{7}$.

3. $\frac{7}{9}$ by $\frac{3}{8}$.

4. $\frac{5}{8}$ by $\frac{4}{9}$.

5. $\frac{7}{9}$ by $\frac{3}{8}$.

6. $\frac{5}{8}$ by $\frac{7}{9}$.

Divide:

7. $\frac{4}{11}$ by $\frac{11}{8}$.

8. $\frac{5}{8}$ by $\frac{7}{4}$.

9. $\frac{6}{15}$ by $\frac{5}{8}$.

10. $\frac{8}{12}$ by $\frac{3}{4}$.

11. $\frac{9}{10}$ by $\frac{3}{8}$.

Divide:

12. $\frac{18}{80}$ by $\frac{6}{10}$.

13. $\frac{7}{25}$ by $\frac{5}{10}$.

14. $\frac{12}{25}$ by $\frac{3}{8}$.

15. $\frac{21}{2}$ by $\frac{7}{8}$.

16. $\frac{86}{49}$ by $\frac{6}{7}$.

17. Divide $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{7}$ by 5 times $\frac{1}{8}$ of $\frac{6}{7}$ of $\frac{1}{5}$.

PROCESS.

$$\frac{2}{3} \times \frac{3}{4} \times \frac{5}{7} \div \frac{5}{1} \times \frac{1}{8} \times \frac{6}{7} \times \frac{1}{5} =$$

$$\frac{2}{3} \times \frac{3}{4} \times \frac{5}{7} \times \frac{1}{5} \times \frac{6}{1} \times \frac{7}{8} \times \frac{5}{1} = \frac{5}{2} = 2\frac{1}{2}$$

EXPLANATION.—In solving

examples like this, all integers and mixed numbers should be changed to improper fractions, and all

the fractions that are factors of the divisor should be inverted. Then find the product of the numerators for the numerator of the quotient, and the product of the denominators for the denominator of the quotient.

When possible use cancellation.

18. Divide $\frac{3}{8}$ of $\frac{5}{7}$ of $\frac{8}{9}$ of 8 by $\frac{2}{3}$ of $\frac{4}{5}$ of 6 .
19. Divide $\frac{5}{8}$ of $\frac{1}{11}$ of $\frac{3}{4}$ of 22 by $\frac{3}{7}$ of $\frac{5}{8}$ of $\frac{7}{9}$ of 10.
20. Divide $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{7}{9}$ of 3 by $\frac{2}{3}$ of $\frac{8}{9}$ of $\frac{1}{10}$.
21. Divide $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{9}$ of 9 by $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of 2.
22. Divide $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of 4 by $\frac{1}{3}$ of $\frac{7}{8}$ of $\frac{9}{10}$ of 3.
23. Divide $\frac{4}{5}$ of $\frac{3}{7}$ of $\frac{5}{8}$ of $2\frac{1}{3}$ by $\frac{6}{8}$ of $\frac{5}{6}$ of $\frac{3}{4}$ of 4.
24. Divide $\frac{3}{8}$ of $\frac{1}{17}$ of $\frac{6}{12}$ of $\frac{1}{5}$ by $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of 7.
25. If a man pays $\$1\frac{1}{3}$ per day for his board, for how long a time will $\$25$ pay for it?
26. How many pounds of beef at $11\frac{1}{2}$ cents per pound can be purchased for $87\frac{1}{2}$ cents?
27. How many acres of land at $\$43\frac{1}{2}$ per acre can be bought for $\$237\frac{1}{2}$?
28. When flour is $\$7\frac{3}{8}$ per barrel, how many barrels can be bought for $\$59$?
29. How many plows at $\$11\frac{1}{5}$ apiece can be bought for $\$67\frac{1}{5}$?
30. How many books at $\$3\frac{1}{8}$ per volume can be purchased for $\$31\frac{1}{4}$?
31. How many diaries at $37\frac{1}{2}$ cents each can be bought for $262\frac{1}{2}$ cents?
32. When oranges are $12\frac{1}{2}$ cents per dozen, how many dozen can be bought for $93\frac{3}{4}$ cents?

182. Expressions of unexecuted division of fractions are sometimes written in the *form of a fraction*.

Such expressions are sometimes called *Complex Fractions*.

Thus, $\frac{3}{8} \div \frac{5}{7}$ may be written $\frac{\frac{3}{8}}{\frac{5}{7}}$; $4\frac{1}{2} \div \frac{3}{8}$, $\frac{4\frac{1}{2}}{\frac{3}{8}}$.

33. Find the value of the fractional form $\frac{\frac{3}{8}}{\frac{5}{7}}$.

PROCESS.

$$\frac{\frac{3}{8}}{\frac{5}{7}} = \frac{3}{8} \div \frac{5}{7} = \frac{3}{8} \times \frac{7}{5} = \frac{21}{40}.$$

Find the value of the following:

34. $\frac{\frac{2}{3}}{\frac{8}{9}}$. 37. $\frac{4}{5\frac{1}{2}}$. 40. $\frac{2\frac{1}{4}}{6\frac{1}{2}}$. 43. $\frac{\frac{3}{4} \text{ of } \frac{3}{8}}{6\frac{1}{4}}$.
35. $\frac{\frac{3}{7}}{\frac{6}{8}}$. 38. $\frac{\frac{3}{8}}{4\frac{1}{2}}$. 41. $\frac{\frac{2}{3}}{\frac{1}{2} \text{ of } 9}$. 44. $\frac{2\frac{1}{7}}{\frac{4}{5} \text{ of } 8}$.
36. $\frac{\frac{5}{8}}{\frac{6}{7}}$. 39. $\frac{5}{\frac{3}{7}}$. 42. $\frac{5\frac{2}{3}}{\frac{2}{3} \text{ of } 5}$. 45. $\frac{1\frac{3}{5} \times 6}{\frac{2}{3} \text{ of } 6}$.

183. A number and its relation to another number given, to find the other number.

ORAL EXERCISES.

1. \$2 is $\frac{1}{2}$ of how many dollars? 3 cents $\frac{1}{3}$ of how many cents?

2. 8 pounds is $\frac{1}{3}$ of how many pounds? 8 pounds $\frac{1}{3}$ of how many pounds?

3. A man spent \$8 for shoes, which was $\frac{1}{6}$ of all the money he had. How much money had he?

4. A lad earned 5 cents per hour, which was $\frac{1}{3}$ as much as his father earned. How much did his father earn?

5. A grocer sold 6 gallons of molasses, which was $\frac{1}{3}$ the quantity the cask contained. How much did the cask contain?

6. In an orchard 10 apple-trees died, which was $\frac{1}{10}$ of all the apple-trees in the orchard. How many apple-trees were there?

7. If 12 is 2-thirds of a number, how much is $\frac{1}{3}$ of the number?

8. If 18 is 6-sevenths of a number, how much is $\frac{1}{7}$ of the number? If 3 is $\frac{1}{7}$ of the number, how much is the number?

9. If 20 is $\frac{4}{5}$ of a number, how much is $\frac{1}{5}$ of the number? If 5 is $\frac{1}{5}$ of the number, how much is the number?

10. Of what number is 4 two-thirds?
 11. 8 is $\frac{4}{5}$ of what number? 6 is $\frac{3}{7}$ of what number?
 12. 9 is $\frac{3}{8}$ of what number? 8 is $\frac{5}{6}$ of what number?
 13. Henry earned \$9, which was $\frac{3}{8}$ of what James earned. How much did James earn?
 14. Mary solved 10 examples, which was $\frac{5}{7}$ of the number her brother solved. How many did her brother solve?

WRITTEN EXERCISES.

184. 1. 240 is $\frac{3}{8}$ of what number?

PROCESS. ANALYSIS.—Since 240 is $\frac{3}{8}$ of a certain number,
 $\frac{3}{8} = 240$ 1-ninth of the number is $\frac{1}{8}$ of 240, or 30; and
 $\frac{1}{8} = 30$ since 30 is $\frac{1}{8}$ of the number, the number must be
 Number = 270 9 times 30, or 270. Hence 240 is $\frac{3}{8}$ of 270.

Find the number of which:

- | | | |
|---------------------------|------------------------------|--|
| 2. 60 is $\frac{3}{7}$. | 7. 120 is $\frac{4}{5}$. | 12. $\frac{4}{5}$ is $\frac{1}{3}$. |
| 3. 45 is $\frac{5}{12}$. | 8. 108 is $\frac{9}{10}$. | 13. $\frac{8}{9}$ is $\frac{2}{3}$. |
| 4. 72 is $\frac{8}{15}$. | 9. 110 is $\frac{11}{12}$. | 14. $\frac{12}{17}$ is $\frac{3}{5}$. |
| 5. 81 is $\frac{9}{14}$. | 10. 132 is $\frac{12}{13}$. | 15. $\frac{14}{15}$ is $\frac{7}{8}$. |
| 6. 65 is $\frac{5}{11}$. | 11. 144 is $\frac{12}{17}$. | 16. $\frac{20}{21}$ is $\frac{5}{7}$. |
17. If \$40 is $\frac{5}{8}$ of my money, how much money have I?
 18. I sold a horse for \$120, which was $\frac{4}{5}$ of what he cost me. How much did he cost me?
 19. A lad earned \$35, which was $\frac{5}{13}$ of what his father earned in the same time. How much did his father earn?
 20. A freight-train ran 15 miles per hour, which was $\frac{3}{8}$ as fast as an express-train. At what rate did the express-train run?
 21. A man owned a farm of 160 acres, which was $\frac{4}{5}$ of the size of his brother's. How many acres were there in his brother's farm?

22. A grocer's profits from the sale of butter were \$240, which was $\frac{8}{15}$ of his profits on vegetables. What were his profits on vegetables?

REVIEW.

ORAL EXERCISES.

185. 1. William had $\$1\frac{1}{2}$, James $\$2\frac{3}{4}$, and Henry $\$3\frac{1}{4}$. How much had they all?

2. A man owned three lots. The first containing $\frac{1}{2}$ acre, the second $\frac{1}{4}$ of an acre, and the third $\frac{1}{8}$ of an acre. What part of an acre was there in all the lots?

3. In $\$5\frac{3}{4}$ how many fourths of a dollar are there?

4. Change $6\frac{1}{8}$ to an improper fraction. $5\frac{3}{7}$. $4\frac{2}{5}$. $6\frac{3}{4}$.

5. Change $\frac{1}{8}$, $\frac{10}{4}$, $\frac{15}{8}$, $\frac{64}{8}$, $\frac{52}{8}$, $\frac{23}{8}$, $\frac{54}{9}$, $\frac{61}{10}$ to integers or mixed numbers.

6. Henry earned the following sums in four successive weeks: $\$5\frac{1}{4}$, $\$4\frac{3}{4}$, $\$6\frac{1}{2}$, $\$3\frac{3}{4}$. How much did he earn?

7. I sold a cow for $\$31\frac{1}{4}$ which cost me $\$28\frac{1}{2}$. How much did I gain?

8. Two men engaged to mow a field of 18 acres. A mowed $5\frac{7}{8}$ acres, and B mowed the rest. How much did B mow?

9. If I spend $\frac{5}{7}$ of my money, what part of it have I left?

10. I gave $\$15\frac{1}{2}$ for a watch and had $\$10\frac{3}{4}$ left. How much money had I at first?

11. After paying $\$20\frac{3}{4}$ for a suit of clothes, Samuel found that he had $\$5\frac{3}{4}$ left. How much had he at first?

12. A man who had \$50, paid a debt of $\$18\frac{1}{2}$, bought books for \$5, and clothes for \$10. How much had he left?

13. A man sold a cow for \$36, which was $\frac{4}{5}$ of what she cost him. How much did she cost him?

14. I sold a cow which cost me \$45 for $\frac{4}{5}$ of her cost. How much did I receive for her?

15. After paying \$48 for some goods, I found that I had but $\frac{1}{7}$ of my money left. How much had I at first?

16. A man sold a watch for \$50, which cost him only $\frac{4}{5}$ of that sum. How much did he gain by the sale?

17. A book-case which cost me \$48 was sold for $\frac{5}{8}$ of its cost. For how much did it sell?

18. What will 4 pairs of pantaloons cost at \$5 $\frac{1}{4}$ a pair?

19. I paid \$20 for hats at \$3 $\frac{1}{3}$ apiece. How many did I buy?

20. I bought 3 pieces of ribbon, each containing 6 $\frac{3}{4}$ yards. How many yards did I buy?

21. If a horse travels 40 $\frac{1}{2}$ miles in 6 hours, at what rate does he travel per hour?

22. A man who had 56 $\frac{3}{4}$ acres of land sold $\frac{3}{8}$ of it. How many acres did he sell? How many acres had he left?

23. A man paid \$40 for a cow. $\frac{3}{8}$ of this sum was $\frac{1}{5}$ of what he paid for a horse. How much did he pay for the horse?

24. After selling $\frac{3}{8}$ of my land, I had 50 acres left. How many acres had I at first?

25. After spending $\frac{1}{8}$ of my money, I found I had \$35 left. How much money had I at first?

26. William found that $\frac{1}{2}$ of his money was $\frac{1}{4}$ of Henry's. What part of Henry's would the whole of William's have been?

27. One-half of A's age was $\frac{4}{5}$ of B's. How many times B's age was A's?

28. If $\frac{1}{3}$ of what James earned per week was $\frac{1}{4}$ of what David earned, how many times what James earned did David earn?

29. If David earned \$4 per week more than James, how much did each earn?

30. $\frac{5}{8}$ multiplied by another number gives a product of $\frac{3}{8}$. What is the multiplier?

31. The difference between $\frac{1}{2}$ of a number and $\frac{1}{4}$ of it is 5. What is the number?

32. James lost $\frac{2}{3}$ of his marbles, and then bought $\frac{1}{3}$ as many as he had left, when he found he had 40 marbles. How many had he at first?

WRITTEN EXERCISES.

186. 1. Find the sum of $6\frac{5}{8}$, $9\frac{5}{12}$, and $18\frac{1}{2}$.

2. A man paid $\$7\frac{1}{2}$ for a barrel of flour, $\$4\frac{1}{4}$ for a cord of wood, and $\$6\frac{3}{8}$ for a ton of coal. How much did they all cost him?

3. How much is $3\frac{1}{4} + 5\frac{1}{8} + 3\frac{3}{8} - 4\frac{1}{8} - 2\frac{1}{4} - 5\frac{1}{8}$?

4. Find the value of $6\frac{1}{4} - 3\frac{1}{8} + 2\frac{1}{8} - 6 + \frac{3}{4} + 1\frac{1}{8}$.

5. Find the value of $\frac{3}{4} \times (2\frac{1}{2} + 3\frac{1}{4}) - \frac{1}{2} \times (4\frac{1}{8} - 3\frac{3}{4})$.

6. Find the value of $(8\frac{1}{4} + 2\frac{1}{8}) \div 3\frac{1}{4} + 3\frac{1}{8} - 2$.

7. Find the value of $(4\frac{1}{8} - 3\frac{1}{8}) \div (4\frac{1}{8} - 3\frac{5}{8}) + \frac{3}{8}$.

8. A farmer exchanged $23\frac{1}{2}$ cords of wood, at $\$4$ per cord, for flour at $\$7$ per barrel. How much flour did he receive?

9. How much will 5 men earn in $5\frac{1}{4}$ days at $\$3\frac{1}{2}$ per day?

10. Mr. Purdy bought 155 acres of land, which was $\frac{1}{10}$ more than he had before he made the purchase. How many acres had he at first?

11. Mr. Reed sold $\frac{3}{4}$ of his farm for $\$4826$. What would the whole farm be worth at that rate?

12. After expending $\$200$ in repairing his house, a man found that he had only $\frac{1}{4}$ of his money left. How much had he at first?

13. A traveler spent $\frac{1}{3}$ of his life in Europe, $\frac{1}{4}$ of it in Asia, and the rest in America. What part of his life did he spend in America?

14. If he lived 25 years in America, how old was he?
15. A man, after spending $\frac{1}{4}$ of his money for a house, and $\frac{1}{5}$ of it for furniture, found that he had \$8100 left. How much had he at first?
16. A man bequeathed $\frac{1}{3}$ of his property to his son, $\frac{1}{5}$ to his brother, and the rest, which was \$9000, to an asylum. What was the value of his property?
17. A merchant sold $\frac{3}{8}$ of a piece of cloth to one lady, $\frac{1}{4}$ of it to another, and the rest, which was 8 yards, to another. How many yards were there in the piece?
18. If A can do a piece of work in 6 days, what part of it can he do in 1 day? If B can do it in 5 days, what part of it can he do in 1 day? If both work together, what part of it can they do in 1 day? If both together can do $\frac{11}{10}$ of the work in 1 day, how long will it take them to do the whole work?
19. A can do a piece of work in 4 days and B can do it in 3 days. In what time can both together do it?
20. A can do a piece of work in 9 days and B can do it in 10 days. In what time can both together do it?
21. A can mow a field in 9 days. A and B together can mow it in 5 days. How long will it take B to mow the entire field alone?
22. Henry and Hiram can together plow a field in 8 days which Henry can plow in 12 days. In what time can Hiram plow it?
23. James had $\frac{3}{4}$ as much money as his brother, and both had \$400. How much had each?
24. Two men together earned \$870. If one earned $\frac{3}{4}$ as much as the other, how much did each earn?
25. In a public school the number of girls was $1\frac{1}{2}$ times the number of boys, and the whole number of pupils was 300. How many boys and how many girls were there in the school?

26. A man who had spent $\frac{1}{2}$ of his money and \$3 more, found that he had \$37 left. How much money had he at first?

27. A and B together can do a piece of work in 10 days. Assuming that A can do but $\frac{1}{2}$ as much as B, in what time can each do the work? -

28. A man who owned $\frac{3}{8}$ of a shoe-factory sold $\frac{2}{3}$ of his share for \$12000. What would the factory be worth if sold at that rate?

29. Find the sum and the difference of 20 and $2\frac{1}{2}$; divide the sum by the difference; the difference by the sum; and find the product of the quotients.

30. A lad lost $\frac{4}{5}$ of his marbles. After buying 6, he found that he had but $\frac{7}{10}$ as many as he had at first. How many had he at first?

QUESTIONS FOR REVIEW.

187. What kind of fractions only can be added? What must be done to dissimilar fractions before they can be added? How should mixed numbers be added? What kind of fractions only can be subtracted? What must be done to dissimilar fractions before they can be subtracted? How should mixed numbers be subtracted?

How may a fraction be multiplied? What is the rule for multiplying a fraction by a fraction? Show that this rule includes the previous cases in multiplication of fractions. What is a compound fraction? How may a fraction be divided? Give the rule for dividing a fraction by a fraction. Show that this rule includes the previous cases in division of fractions. What is a complex fraction?

What is a fraction? What is the numerator? What is the denominator? What is reduction of fractions? How are fractions reduced to their smallest terms? How are mixed numbers reduced to improper fractions? How are improper fractions changed to integers or mixed numbers? How are dissimilar fractions made similar? What is meant by the least common denominator of fractions?

DECIMAL FRACTIONS

188. 1. If a line is divided into 10 equal parts, what is each part called?

2. If $\frac{1}{10}$ of a line is divided into 10 equal parts, what part of the whole line is each part? How much is $\frac{1}{10}$ of $\frac{1}{10}$?

3. If $\frac{1}{100}$ of a line is divided into 10 equal parts, what part of the line is each part? How much is $\frac{1}{10}$ of $\frac{1}{100}$?

4. How many hundredths are equal to $\frac{1}{10}$?

5. How many thousandths are equal to $\frac{1}{100}$?

6. The divisions of any thing into tenths, hundredths, thousandths, etc., are called *Decimal* divisions.

189. A **Decimal Fraction** is one or more of the decimal divisions of a unit.

The word *decimal* is derived from the Latin word *decem*, which means *ten*.

Decimal fractions are usually called *decimals*.

190. Since decimals have the same law of increase and decrease as integers, the *denominator* may be indicated by the *position* of the figures. Hence:

191. The figures in the *first place* at the right of units represent *tenths*; in the *second*, *hundredths*; in the *third*, *thousandths*; in the *fourth*, *ten-thousandths*; etc.

192. The **Decimal Point** is a period placed before the decimal.

Thus .6 represents $\frac{6}{10}$; .24 represents $\frac{24}{100}$.

The decimal point is also called the *Separatrix*, inasmuch as it is used to separate integers from decimals.

.6 represents	$\frac{6}{10}$.	.08 represents	$\frac{8}{100}$.	.005 represents	$\frac{5}{1000}$.
.5	" $\frac{5}{10}$.	.04	" $\frac{4}{100}$.	.006	" $\frac{6}{1000}$.
.9	" $\frac{9}{10}$.	.09	" $\frac{9}{100}$.	.007	" $\frac{7}{1000}$.
.7	" $\frac{7}{10}$.	.07	" $\frac{7}{100}$.	.008	" $\frac{8}{1000}$.

NUMERATION TABLE.

Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
3	4	5	6	3	2	1	7	4
INTEGERS.			DECIMALS.					

The orders below millionths in their order are: ten-millionths, hundred-millionths, billionths, ten-billionths, hundred-billionths, etc.

193. What order of decimals occupies:

1st place.

2d place.

7th place.

3d place.

4th place.

2d place.

5th place.

6th place.

4th place.

What decimal place is occupied by:

Tenths.

Ten-thousandths.

Ten-millionths.

Thousandths.

Millionths.

Hundredths.

Hundredths.

Hundred-thousandths.

Thousandths.

EXERCISES IN NUMERATION.

194. 1. Read 42.356.

ANALYSIS.—The decimal part of the number expresses 3 tenths, 5 hundredths, 6 thousandths, or 356 thousandths. The whole expression is therefore read: 42 and 356 thousandths.

RULE.—*Read the decimal as integer and give it the denomination of the right-hand figure.*

In reading an integer and decimal, use the word *and* only between the integral and decimal parts of the number.

Read the following:

2. .29.	9. 2.29.	16. 216.345.
3. .47.	10. 3.46.	17. 315.0042.
4. .376.	11. 4.056.	18. 26.04321.
5. .045.	12. 5.732.	19. 32.00046.
6. .0468.	13. 16.714.	20. 30.000073.
7. .4235.	14. 29.043.	21. 300.000006.
8. .6824.	15. 32.0042.	22. 260.000426.

EXERCISES IN NOTATION.

195. 1. Express decimally 29 thousandths.

ANALYSIS.—Since 29 thousandths is equal to 2 hundredths and 9 thousandths, 9 is written in thousandths' place, 2 in hundredths' place, and since there are no tenths, 0 in tenths' place. We then place a decimal point before the tenths.

Hence 29 thousandths = .029.

RULE.—*Write the numerator of the decimal, prefix ciphers, if necessary, to indicate the denominator, and place the decimal point before tenths.*

The number of places in a decimal will be equal to the number of ciphers in the denominator.

Express decimally :

2. Nine tenths. Eight tenths. Seven tenths.
3. Fourteen hundredths. Twenty-one hundredths.
4. Sixteen thousandths. Two hundred two thousandths.
5. Nine ten-thousandths. Eighty-four millionths.
6. Nine and two hundred sixteen hundred-thousandths.
7. Eleven and six hundred twenty-two thousandths.
8. Forty and ninety-six ten-thousandths.
9. Thirty-seven and forty-four hundred-thousandths.
10. Seventy-three and eight millionths.
11. 426 hundred-thousandths. 36 millionths.
12. 729 ten-thousandths. 324 thousandths.
13. 406 millionths. 370 ten-thousandths.
14. 35 hundred-thousandths. 310 thousandths.
15. 215 ten-millionths. 14 billionths.
16. $\frac{265}{1000}$.
17. $\frac{21}{10000}$.
18. $5\frac{85}{1000}$.
19. $4\frac{18}{100}$.
20. $\frac{889}{10000}$.
21. $216\frac{884}{1000000}$.
22. $283\frac{81}{100000}$.
23. $165\frac{4}{1000000}$.
24. $239\frac{5}{100}$.
25. $416\frac{75}{10000}$.
26. $329\frac{814}{1000000}$.
27. $1864\frac{81}{10000000}$.

REDUCTION OF DECIMALS.

196. To reduce dissimilar decimals to similar decimals.

1. Read the decimals .8 ; .80 ; .800.
2. How does $\frac{8}{10}$ compare in value with $\frac{80}{100}$? With $\frac{800}{1000}$?
3. Read the decimals .43 ; .430 ; .4300.
4. How does $\frac{43}{100}$ compare in value with $\frac{430}{1000}$? With $\frac{4300}{10000}$?
5. How does annexing a cipher to a decimal affect its value ?

197. PRINCIPLE.—*Annexing a cipher to a decimal does not alter its value.*

WRITTEN EXERCISES.

198. 1. Reduce .4, .36, and .035 to similar fractions.

PROCESS. **ANALYSIS.**—Since the lowest order of decimals is thousandths, all the decimals must be changed to thousandths.
 $.4 = .400$
 $.36 = .360$
 $.035 = .035$
 Since annexing ciphers to a decimal does not alter its value (Prin.), we give to each number three decimal places by annexing ciphers, thus making them similar.

RULE.—*Give to all the decimals the same number of places by annexing ciphers.*

Reduce the following to similar decimals:

- | | |
|------------------------|-------------------------|
| 2. .58, .023, .0465. | 7. 3.5, .462, 32.0457. |
| 3. .3, .0425, .00465. | 8. 2.9, 4.13, .23045. |
| 4. .05, .004, .004631. | 9. .61, 31.46, .41304. |
| 5. .003, .042, .00345. | 10. .003, 3.04, 3.0046. |
| 6. .06, .423, .040404. | 11. 8.2, 13, .046135. |

199. To reduce a decimal to a common fraction.

1. If 5 tenths is written as a common fraction, what is the numerator? What is the denominator?
2. What is the numerator and what the denominator of 12 hundredths when expressed as a common fraction?
3. Express the value of the decimal 20 hundredths by a common fraction in its smallest terms.

WRITTEN EXERCISES.

200. 1. Reduce .75 to its equivalent common fraction.

PROCESS. **ANALYSIS.**—.75 expressed as a common fraction is $\frac{75}{100}$, which, being reduced to its smallest terms, is $\frac{3}{4}$.
 $.75 = \frac{75}{100} = \frac{3}{4}$

RULE.—*Omit the decimal point, supply the denominator, and reduce the fraction to its smallest terms.*

Reduce the following to common fractions in their smallest terms:

2. .25.	6. .375.	10. .0375.	14. .0040.
3. .65.	7. .225.	11. .0250.	15. .05625.
4. .35.	8. .875.	12. .0625.	16. .00625.
5. .40.	9. .125.	13. .0750.	17. .00875.

201. To reduce a common fraction to a decimal.

1. How many tenths are there in $\frac{1}{2}$? In $\frac{1}{3}$?
2. How many hundredths are there in $\frac{1}{4}$? In $\frac{1}{5}$?
3. How many hundredths are there in 1? In $\frac{1}{6}$? In $\frac{2}{3}$?
4. How many thousandths are there in 1? In $\frac{1}{8}$? In $\frac{3}{8}$?
5. How many tenths are there in $\frac{1}{5}$? In $\frac{2}{5}$? In $\frac{3}{5}$? In $\frac{4}{5}$?

WRITTEN EXERCISES.

1. Reduce $\frac{5}{8}$ to an equivalent decimal.

PROCESS. **ANALYSIS.**— $\frac{5}{8}$ is $\frac{1}{8}$ of 5, or of 50 tenths; and $\frac{1}{8}$ of 50 tenths is 6 tenths and 2 tenths remaining. 2 tenths are equal to 20 hundredths, and $\frac{1}{8}$ of 20 hundredths is 2 hundredths and 4 hundredths remaining. 4 hundredths are equal to 40 thousandths, and $\frac{1}{8}$ of 40 thousandths is 5 thousandths. Hence $\frac{5}{8}$ is equal to 6 tenths + 2 hundredths + 5 thousandths, or .625.

RULE.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

In many cases the division is not exact. In such instances the remainder may be expressed as a common fraction, or the sign + may be employed after the decimal to show that the result is not complete; thus: $\frac{1}{3} = .166\frac{2}{3}$, or .166 +.

Reduce the following to decimals :

2. $\frac{3}{8}$.	6. $\frac{3}{75}$.	10. $\frac{13}{25}$.	14. $\frac{3}{125}$.
3. $\frac{7}{8}$.	7. $\frac{2}{25}$.	11. $\frac{17}{20}$.	15. $\frac{4}{75}$.
4. $\frac{5}{16}$.	8. $\frac{1}{40}$.	12. $\frac{21}{80}$.	16. $\frac{3}{7}$.
5. $\frac{19}{20}$.	9. $\frac{1}{80}$.	13. $\frac{5}{8}$.	17. $\frac{15}{16}$.

ADDITION OF DECIMALS.

202. 1. What is the sum of $\frac{3}{10}$ and $\frac{7}{10}$? Of .4 and .7?
 2. What is the sum of $\frac{5}{100}$ and $\frac{12}{100}$? Of .08 and .09?
 3. What is the sum of $\frac{1}{10}$ and $\frac{1}{100}$? Of .5 and .25?
 4. What is the sum of .15 and .015? Of .12 and .5?

PRINCIPLES.—*The principles are the same as for addition of common fractions.*

203. 1. What is the sum of 1.36, 3.253, and .0453?

PROCESS.

$$\begin{array}{r} 1.36 = 1.3600 \\ 3.253 = 3.2530 \\ .0453 = .0453 \\ \hline 4.6583 \end{array}$$

ANALYSIS.—The numbers are written so that units stands under units, tenths under tenths, etc.

The decimals may be made similar by annexing ciphers and then added, care being taken to place the decimal point in the sum directly under the decimal point in the numbers added. Or,

Since only units of one order are found in any column, it is unnecessary, in practice, to make them similar.

Find the sum of the following:

- | | | | | | |
|-----------|--------|--------|------------|---------|---------|
| 2. 3.25, | .426, | 38.41. | 9. 2.943, | 31.296, | 4.0005. |
| 3. 1.6, | 32.43, | .0435. | 10. 36.4, | 1.625, | 31.328. |
| 4. 35, | 2.375, | 1.046. | 11. 1.593, | 384.2, | 4.2865. |
| 5. 2.1, | 3.283, | 46.2. | 12. 3.569, | 268, | 31.463. |
| 6. .3568, | 22.9, | .425. | 13. 35.63, | 26.8, | 246.59. |
| 7. 16.28, | 14.35, | 21.73. | 14. .2184, | 21.84, | 218.4. |
| 8. 35.27, | .1264, | 18.43. | 15. 26.43, | 16.875, | 27.34. |

16. What is the sum of 9 hundredths, 45 thousandths, 8 tenths, and 146 thousandths?

17. What is the sum of 354 thousandths, 213 millionths, 3564 hundred-thousandths, 9 tenths, and 18 hundredths?

18. A merchant's sales were as follows: On Monday \$369.34, on Tuesday \$296.18, on Wednesday \$473.39, on Thursday \$468.37. How much were they for those four days?

19. A pedestrian walked 49.13 miles the first day, 33.13 the second day, 46.19 the third day, and 39.47 miles on the fourth day. How far did he travel in the four days?

SUBTRACTION OF DECIMALS.

204. 1. From $\frac{7}{10}$ take $\frac{4}{10}$. From .8 take .3.

2. From $\frac{8}{100}$ take $\frac{5}{100}$. From .12 take .07.

3. From 4 take $\frac{5}{10}$. From 3 take .3.

4. From 6 take $2\frac{5}{10}$. From 7 take 4.3.

PRINCIPLES.—*The principles are the same as for subtraction of common fractions.*

WRITTEN EXERCISES.

205. 1. From 24.23 subtract 8.5624.

PROCESS.

$$\begin{array}{r} 24.23 \\ 8.5624 \\ \hline 15.6676 \end{array} \quad \begin{array}{r} = 24.2300 \\ = 8.5624 \\ \hline 15.6676 \end{array}$$

ANALYSIS.—The numbers are written so that units stand under units, tenths under tenths, etc.

The decimals may be made similar and then subtracted, care being taken to place the decimal point in

the remainder directly under the decimal point in the numbers subtracted.

In the first process the ciphers are written, but they may be supposed to be annexed when we subtract, and consequently need not be written.

	(2.)	(3.)	(4.)	(5.)
From	27.35.	36.38.	84.32.	41.86.
Subtract	<u>15.42.</u>	<u>21.465.</u>	<u>66.375.</u>	<u>22.39.</u>
	(6.)	(7.)	(8.)	(9.)
From	36.42.	27.18.	31.2.	26.
Subtract	<u>13.245.</u>	<u>6.4356.</u>	<u>11.543.</u>	<u>4.572.</u>

10. What is the difference between 6.285 and 3.2846?
11. What is the difference between 4.83 and .4836?
12. What is the difference between 3.28 and .0436?
13. From 65 hundredths subtract 65 hundred-thousandths.
14. From 193 ten-thousandths subtract 1426 millionths.
15. From 97 thousandths subtract 456 ten-thousandths.
16. Find the value of $2.35 - .064 + 3.23 - 1.2356$.
17. Find the value of $3.572 + 2.36 - .425 - 2.6142$.
18. Find the value of $5.23 + .329 - 4.164 - .00045$.
19. Find the value of $4.6 + 2.3506 - 1.004 - 3.3$.
20. Find the value of $3.8004 - 1.00006 + 3.7 - 4.0405$.
21. The receipts of a manufactory were \$18269.25 and the expenses were \$11243.59. What was the gain?
22. A railroad ticket agent sold in 1880 tickets to the value of \$13269.43, and in 1881 to the value of \$17839.54. How much did the sales in 1881 exceed the sales in 1880?

MULTIPLICATION OF DECIMALS.

206. 1. What is the product of $\frac{3}{10} \times 2$? $.4 \times 2$?
2. How many decimal figures are there in the product of tenths by units?
3. What is the product of $\frac{3}{100} \times 4$? $.04 \times 2$?
4. How many decimal figures are there in the product of hundredths by units?
5. What is the product of $\frac{3}{10} \times \frac{3}{10}$? $.4 \times .2$?

6. How many decimal places are required to express the product of tenths multiplied by tenths?

7. What is the product of $\frac{2}{10} \times \frac{8}{100}$? $.4 \times .02$?

8. How many decimal places are required to express the product of tenths multiplied by hundredths? Tenths by thousandths? Hundredths by thousandths?

9. How does the number of places required to express the product of two decimals compare with the number of decimal places in the factors?

207. PRINCIPLE.—*The product of two decimals contains as many decimal places as there are decimal places in both factors.*

WRITTEN EXERCISES.

208. 1. What is the product of .315 multiplied by .23?

PROCESS.

$$\begin{array}{r} .315 \\ \times .23 \\ \hline 945 \\ 630 \\ \hline .07245 \end{array}$$

ANALYSIS.—Since 315 and 23 are the numerators of the fractions, in multiplying the fractions we must multiply 315 by 23, which gives 7245 for a new numerator.

Since thousandths multiplied by hundredths produce hundred-thousandths, the product of the fractions is 7245 hundred-thousandths, or .07245.

Or, The number of decimal places in the product will be five, the number in both factors. (Prin.)

RULE.—*Multiply as if the numbers were integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.*

If the product does not contain a sufficient number of decimal places, the deficiency must be supplied by prefixing ciphers.

Multiply:

2. .32 by .37.

3. .42 by 2.4.

4. .56 by 28.

Multiply:

5. .72 by .305.

6. .82 by .046.

7. .916 by .314.

Multiply:

8. .825 by 2.63.

9. .423 by 3.6.

10. 21.7 by .034.

11. 3.23 by .0056. 15. .812 by .0034. 19. 1.894 by .0234.
 12. .169 by 3.042. 16. 3.46 by .00426. 20. 19.16 by .0046.
 13. 24.3 by 3.75. 17. .2143 by .0042. 21. 23.97 by .00256.
 14. 16.9 by .025. 18. .1693 by .00365. 22. 23.04 by 3.1605.
 23. Multiply 6.832 by 100.

PROCESS. $\begin{array}{r} 6.832 \\ \times 100 \\ \hline 683.200 \end{array}$	ANALYSIS. —Since each removal of a figure one place to the left increases its value tenfold, the removal of the decimal point one place to the right multiplies by 10, and the removal of the point two places multiplies by 100.
---	--

RULE.—*To multiply by 1 with any number of ciphers annexed, remove the decimal point as many places to the right as there are ciphers annexed to the 1.*

24. Multiply 28.63 by 10. 27. Multiply .384 by 100.
 25. Multiply 38.295 by 100. 28. Multiply 5.276 by 10.
 26. Multiply 2.835 by 1000. 29. Multiply .84325 by 10000.
 30. What will 38.25 acres of land cost at \$22.35 per acre?
 31. What will 46.25 bushels of wheat cost at \$1.73 per bushel?
 32. A merchant sold 12.35 yards of silk at \$3.15 per yard. How much did he receive for it?

DIVISION OF DECIMALS.

- 209.** 1. What is the product of .6 by 8?
 2. 4.8 is the product of two factors, one of which is 8: what is the other factor?
 3. What is the product of .6 by .8?
 4. .48 is the product of two factors, one of which is .6: what is the other factor?
 5. What is the product of .06 by .8?

6. .048 is the product of two factors, one of which is .06: what is the other factor?

7. How many decimal places are there in the product of any two factors?

8. If the product and one of two factors are given, how may the number of decimals in the other factor be found?

9. Since the factor sought will be the quotient, how many decimal places will there be in the quotient?

210. PRINCIPLE.—*The quotient will contain as many decimal places as the number of decimal places in the dividend exceeds those in the divisor.*

WRITTEN EXERCISES.

211. 1. Divide .07245 by .23.

PROCESS.

$$\begin{array}{r}
 .23 \overline{) .07245} \quad (.315 \\
 \underline{69} \\
 34 \\
 \underline{23} \\
 115 \\
 \underline{115} \\
 0
 \end{array}$$

ANALYSIS.—Since the dividend is the product of the divisor by the quotient, the numerator of the dividend, 7245, is the product of the numerator of the divisor, 23, by the numerator of the quotient. Dividing 7245 by 23 we obtain 315 for the numerator of the quotient.

The denominator of the dividend divided by the denominator of the divisor gives 1000 for the denominator of the quotient. Hence, the quotient is $\frac{315}{1000}$, or .315.

The number of decimal places in the quotient may be found by subtracting the number of places in the divisor from the number of places in the dividend. (Prin.)

RULE.—*Divide as if the numbers were integers, and from the right of the quotient point off as many figures for decimals as the number of decimal places in the dividend exceeds those in the divisor.*

1. If the quotient does not contain a sufficient number of decimal places the deficiency must be supplied by prefixing ciphers.

2. Before commencing the division, the number of decimal places in the dividend should be made at least equal to the number of decimal places in the divisor by annexing ciphers.

3. When there is a remainder after using all the figures of the dividend, annex decimal ciphers and continue the division.

Divide:

2. .0864 by .24.
3. 1.075 by .43.
4. 1.5652 by .043.
5. .18312 by .056.
6. .12126 by .235.
7. 2.1828 by .034.
8. 16.1262 by 3.06.
9. 1.3621 by 5.14.
10. .016074 by .047.
11. .48912 by 7.34.
12. 167.544 by 71.6.
13. 2.16194 by .317.

Divide:

14. 1.1502 by .0027.
15. .011487 by .0035.
16. 4.75632 by .1468.
17. 45.4723 by .316453.
18. .0342568 by .006523.
19. .308146 by .063725.
20. 4.19289 by .0050723.
21. 3164 by .04683.
22. 328.54 by 2.6837.
23. 413.27 by 6864.5.
24. 23.45 by 26.817.
25. 30000 by .000003.

26. Divide 325.47 by 100.

<p>PROCESS.</p> $\begin{array}{r} 100 \overline{) 325.47} \\ 3.2547 \end{array}$	<p>ANALYSIS.—Since each removal of a figure one place to the right decreases its value tenfold, the removal of the decimal point one place to the left divides by 10, and the removal two places divides by 100.</p>
--	--

RULE.—*To divide by 1, with any number of ciphers annexed, remove the decimal point as many places to the left as there are ciphers annexed to the 1.*

27. Divide 392.5 by 100.
28. Divide 26.45 by 10.
29. Divide 369.5 by 1000.
30. Divide 4.825 by 100.
31. Divide 38.62 by 1000.
32. Divide 42.85 by 10000.
33. If 61 maps cost \$137.25, what is the cost of each?

34. If 4.7 acres of land produce 131.6 bushels of wheat, what is the average crop per acre?

35. If a train moves 15.3 miles per hour, how long will it take to go 362.35 miles?

36. A stove dealer received \$1204.35 for 37 stoves. 'How much did he receive for each?

37. A clothier paid \$238.50 for 75 pairs of trousers. How much did he pay for each pair?

QUESTIONS FOR REVIEW.

212. What is a decimal fraction? From what is the word *decimal* derived? Where are tenths written with reference to units? What is the decimal point? What else is it sometimes called? Why?

What is the rule for numeration of decimals? Where is the word *and* used in reading a decimal? What is the rule for notation of decimals? How does the number of places in a decimal compare with the number of ciphers in the denominator?

How does annexing a cipher to a decimal affect its value? How are dissimilar decimals made similar? How are decimals reduced to common fractions? How are common fractions reduced to decimals?

How are decimals added? How are decimals subtracted? What is the principle relating to multiplication of decimals? What is the rule for multiplication of decimals? If the product does not contain as many figures as there are decimals in both factors, what is done? How do we multiply by 1 with ciphers annexed?

What is the principle relating to division of decimals? What is the rule for division of decimals? If the quotient does not contain a sufficient number of decimal places, what is done? Before commencing division, how should the number of decimal places in dividend and divisor compare? What is done when there is a remainder, after using all the figures of the dividend? How do we divide by 1 with ciphers annexed?

What is a fraction? What is the rule for division of fractions? What is the rule for multiplication of fractions? What is a complex fraction? How may a fraction be multiplied? How may a fraction be divided? What effect has multiplying or dividing the terms of a fraction by the same number?

UNITED STATES MONEY.

213. United States Money is the money of the United States.

Of this money:

10 mills	make	1 cent,	marked	c.
10 cents	make	1 dime,	"	d.
10 dimes	make	1 dollar,	"	\$.
10 dollars	make	1 eagle,	"	E.

214. Coins are stamped pieces of metal, having a value fixed by law.

215. Paper Money is notes and bills issued by the Government and banks, and authorized to be used as money.

The *coins* of the United States are made of *gold*, *silver*, *nickel* and *bronze*.

The principal *gold* coins are: double-eagle, eagle, half-eagle, quarter-eagle, three-dollar piece and dollar piece.

The *silver* coins are: dollar, half-dollar, quarter-dollar, and ten-cent piece.

The *nickel* coins are: the five-cent piece and the three-cent piece.

The *bronze* coins: the cent.

Mills are not coined.

For exercises in Notation and Numeration, see Art. 37.

ORAL EXERCISES.

216. 1. How many cents are there in 20 mills? In 40 mills?

2. How many dimes are there in 30 cents? In 50 cents?

3. How many dollars are there in 30 dimes? In 60 dimes?

4. How many cents are there in 1 dollar? In 4 dollars?

5. How many dollars are there in 200 cents? In 500 cents?

6. How many eagles are there in 20 dollars? In 40 dollars?

7. How many mills are there in $\frac{1}{2}$ cent? In $\frac{1}{4}$ cent?

8. How many cents are there in $\frac{1}{2}$ dollar? In $\frac{1}{4}$ dollar?
In $\frac{1}{8}$ dollar? In $\frac{1}{16}$ dollar?

9. How many cents are there in $\frac{1}{8}$ of a dollar? In $\frac{3}{8}$ of a dollar? In $\frac{5}{8}$ of a dollar? In $\frac{7}{8}$ of a dollar?

10. What part of a dollar are 10 cents? 20 cents? 50 cents? 75 cents?

11. What part of a dollar are 25 cents? 30 cents? 60 cents?

12. How many mills are there in one dollar?

WRITTEN EXERCISES.

217. Since 10 mills make 1 cent, 100 cents make a dollar, and 1000 mills make a dollar, we have the following rules:

RULES.—1. *Cents are changed to mills by multiplying by 10; that is, by annexing one cipher to the number.*

2. *Mills are changed to cents by dividing by 10; that is, by pointing off one figure from the right of the number.*

3. *Dollars are changed to cents by multiplying by 100; that is, by annexing two ciphers to the number.*

4. *Cents are changed to dollars by dividing by 100; that is, by placing the decimal point between the second and third figures on the right of the number.*

5. *Dollars are changed to mills by multiplying by 1000; that is, by annexing three ciphers to the number.*

6. *Mills are changed to dollars by dividing by 1000; that is, by placing the decimal point between the third and fourth figures on the right of the number.*

1. Change 46 cents to mills.
2. Change 32 mills to cents.
3. Change 35 cents to mills.
4. Change 46 mills to cents.
5. Change 36 dollars to cents.
6. Change 24 dollars to cents.
7. Change 185 dollars to cents.
8. Change 215 dollars to cents.
9. Change 4500 cents to dollars.
10. Change 800 cents to dollars.
11. Change 725 cents to dollars.
12. Change 1437 cents to dollars.
13. Change 15 dollars to mills.
14. Change 8 dollars to mills.
15. Change 1600 mills to dollars.
16. Change 1845 mills to dollars.
17. Change 2437 mills to dollars.
18. Change 60 dollars and 30 cents to cents.
19. How many cents are there in \$16.25?
20. How many cents are there in \$18.93?
21. How many cents are there in \$45.27?
22. How many mills are there in \$6.755?
23. How many dollars are there in 4684 cents?
24. How many dollars are there in 5483 cents?
25. How many dollars are there in 6873 cents?

ORAL EXERCISES.

218. 1. James had 50 cents and his brother gave him 50 cents more. How much had he then?
2. Mr. Goode divided \$1.20 equally among 4 boys. How much did each receive?
3. A man who had \$5 paid \$1.50 for flour. How much had he left?

4. A mechanic earned \$2.50 per day. How much did he earn in 6 days?

5. A boy who had 35 cents spent 5 cents for marbles, 10 cents for a whip, and 10 cents for a writing-book. How much had he left?

6. How much will 4 books cost at 50 cents each?

7. If I buy a book for $\frac{1}{2}$ dollar and paper for $\frac{1}{4}$, and give the stationer 1 dollar, how many cents should I receive in change?

8. How much less than 50 cents will $\frac{5}{7}$ of a gallon of molasses cost at 35 cents per gallon?

9. A man bought 4 pounds of butter at 20 cents a pound and 4 bars of soap at 10 cents a bar. How much did both cost?

10. When flour is worth \$9 per barrel, what will $\frac{1}{4}$ of a barrel cost?

11. How many yards of cloth at 30 cents per yard can be bought for \$1.50?

12. How much must I pay for 3 pounds of tea at 60 cents per pound?

13. If a grocer sells tea for 60 cents per pound that cost him 40 cents, how much is his gain on 10 pounds?

14. A lad earned 20 cents per day. How much did he earn in 10 days?

15. A lad wishing to buy a pair of skates that cost \$2.50 found that he had but $\frac{1}{2}$ dollar in silver. He afterwards earned \$1.50. How much did he still lack of having money enough to make the purchase?

16. An apple-vender sold 5 apples at 2 cents each and 6 apples at 3 cents each. If they cost $\frac{1}{4}$ of a dollar, how much did he gain?

17. A girl who had $\frac{1}{2}$ dollar and $\frac{1}{4}$ dollar spent 15 cents for needles, 10 cents for thread, 20 cents for braid, and 30 cents for knitting materials. How much had she left?

WRITTEN EXERCISES.

219. 1. A man owes the following sums: for groceries \$69.25; for clothing \$293.74; for books \$161.23. How much does he owe?

2. Find the sum of 36 dollars 8 cents, 46 dollars 7 cents, 18 dollars 32 cents, 9 dollars 16 cents, and 4 dollars 32 cents.

3. Find the sum of 18 dollars 6 cents 4 mills, 19 dollars 8 cents 8 mills, 13 dollars 12 cents 5 mills.

4. A lawyer collected the following sums: \$26.94; \$129.38; \$17.94; \$289.07; \$13.65. How much did he collect?

5. Mr. Rutherford paid the following bills for repairs upon his premises, viz: carpenter-work \$313.29; plastering \$169.37; plumbing \$64.86; other expenses \$104.39. How much did he pay for repairs?

6. A farmer received for his wheat \$296.83; for his oats \$110.39; for his barley \$46.83; and for his corn \$394.89. How much did he receive for all?

7. A man sold 43.5 acres of land which cost him \$38 50 per acre for \$2000. How much did he gain?

8. A drover purchased 318 head of cattle for \$10000. If he sold them at the average price of \$32.15, did he gain or lose by the sale, and how much?

9. A farmer sold 41.25 pounds of butter for \$6.85. How much did he receive a pound for it?

10. If 5.3 yards of cloth cost \$9.275, what will 8.5 yards cost?

11. If 15 tons of hay cost \$125.25, what will 12 tons cost?

12. How many pairs of boots at \$3.25 can be bought for \$169?

13. A man worked 9 days for $\$2.12\frac{1}{2}$ per day. How much did he earn?

14. A drover bought 375 sheep at $\$4.25$ per head. If he sold them at a gain of $\$75$, how much did he get per head for them?

15. What will be the cost of 4 pairs of gloves at $\$1.75$ a pair, 6 yards of ribbon at $\$.40$ per yard, 18 yards of muslin at 12 cents per yard, and 4 pairs of hose at $\$.62\frac{1}{2}$ a pair?

16. A man earned $\$125$ in 40 days by working 10 hours per day. How much did he earn per hour?

17. Mr. Scribe owed me $\$500$. He gave in payment 565 bushels of corn at 45 cents per bushel and the rest in money. How much money did he pay me?

18. If I earn $\$60$ per month and spend $\$35.50$ of it, in how many months will I save $\$1000$?

19. A merchant sold 25.5 yards of cambric at $\$.20$ per yard and gained $\$1.70$. How much did it cost him?

20. The gain of a manufactory was $\$8269.35$. If it was owned by 9 equal partners, what was each man's share of the gain?

21. A grocer bought 8 barrels of sugar, each containing 235 pounds, for $\$122.20$. How much did it cost per pound?

22. If 34.25 reams of paper cost $\$54.80$, what is the cost per ream?

23. The expenses of an agent were $\$23.25$ per week. If his income was $\$50$ per week, how much would he save in 13 weeks?

24. During the war of the rebellion a paper dollar was worth at one time only 65 cents in gold. How many dollars in gold would 520 dollars in paper money buy at that time?

25. A man who had an income of $\$3215$, spent $\frac{1}{3}$ of it for household expenses and $\frac{1}{3}$, the remainder, in traveling. How much did he spend in traveling?

ACCOUNTS AND BILLS.

220. A **Debt** is an amount which one person owes to another.

221. A **Credit** is an amount which is due to a person, or a sum paid towards discharging a debt.

222. A **Debtor** is a party owing a debt.

223. A **Creditor** is a party to whom a debt is due.

224. An **Account** is a record of debts and credits between parties doing business with each other.

225. The **Balance of an Account** is the difference between the amount of the debts and credits.

226. A **Bill** is a written statement given by the seller to the buyer, of the quantity and price of each article sold, and the amount of the whole.

227. The **Footing of a Bill** is the total cost of all the articles.

228. A bill is **Receipted** when the words *Received Payment* are written at the bottom, and the creditor's name is signed either by himself or some authorized person.

229. The following abbreviations are in common use:

@,	At.	Do.,	The same.	Mdse.,	Merchandise.
%,	Account.	Doz.,	Dozen.	No.,	Number.
Acc't,	Account.	Dr.,	Debtor.	Pay't,	Payment.
Bal.,	Balance.	Fr't,	Freight.	Pd.,	Paid.
Bbl.,	Barrel.	Hhd.,	Hogshead.	Per,	By.
Bo't,	Bought.	Inst.,	This month.	Rec'd,	Received.
Co.,	Company.	Int.,	Interest.	Yd.,	Yard.
Cr.,	Creditor.	Lb.,	Pound.	Yr.,	Year.

(1. Cincinnati, O., May 25, 1882.

Mr. Howard B. Judson,

Bought of Smith & Williams:

3 bbl. Coffee Sugar (A)	647 lb., @ \$	$.11\frac{1}{4}$	\$ 72.79
5 chests Oolong Tea	255 " "	.81	206.55
8 " Black "	167 " "	.67	111.89
7 boxes Cheese	801 " "	$.18\frac{1}{2}$	40.68

Received Payment,

\$481.86

Smith & Williams,
Per B. B.

Copy, fill out, and find the footings of the following:

(2.)

ROCHESTER, *March 1, 1882.*

MR. J. B. ADAMS,

Bought of HOWE & ROGERS:

75½ yards of Carpeting	@	\$2.12½	\$
37 yards of Drugget	"	1.20	
8 Rugs	"	4.16	
5 Mats	"	2.37½	
18 yards Oil-cloth	"	1.08	
9 yards Carpet Lining	"	.12½	
			\$

(3.)

MEMPHIS, *May 20, 1882.*

MR. GEORGE B. SHERMAN,

To SAMUEL B. SMALLWOOD, Dr.

To 37 bbl. Pork	@	\$24.35	\$
" 127 bbl. Flour	"	8.15	
" 3 hhd. Molasses—169 gal.	"	.43	
" 29 firkins Butter—2120 lb.	"	.31	
" 3 boxes Raisins	"	4.65	
" 5 bbl. Kerosene—207 gal.	"	.18½	
			\$

(4.)

NEW YORK, *April 1, 1882.*

MR. HENRY C. BROWN,

To STURDEVANT & Co., Dr.

1882.							
Jan.	9	To 3 Gold Watches—\$124.50, \$61.24, \$57.18				\$	
"	13	" 437 pwt. Gold Chain	@	1.15			
Feb.	3	" 35 sets Plated Tea-service.	"	43.10			
"	15	" 7 " " "	"	51.			
Mar.	8	" 5 Silver Pie-knives	"	12.			
"	12	" 12 Plated Ice-pitchers	"	12.50			
						\$	

Make out in proper form and receipt the following:

5. Mrs. M. T. Dean bought of G. C. Smith & Co. 25 yd. of calico @ 10 cents, 37 yd. of sheeting @ $18\frac{1}{4}$ cents, 2 pairs of gloves @ \$1.50, 1 sun-umbrella @ \$6.75, 5 yd. of Hamburg edging @ 25 cents, 6 handkerchiefs @ $37\frac{1}{2}$ cents, and 7 pairs of hose @ 85 cents.

6. Mr. Hubert Stone bought of Henry Dearborn & Co. 125 bbl. flour @ \$6.90, 115 bushels of wheat @ \$1.20, 324 bushels of corn @ \$.58, 675 bushels of oats @ \$.40, and 50 bbl. potatoes @ \$1.25.

7. Henry Easton & Co. bought of Barnes & Bancroft 24 yards of cloth @ \$1.36, 18 yards of velvet @ \$6.25, 29 yards of silk at \$2.75, 36 yards ribbon @ \$.75, and 19 yards satin at \$4.35.

8. James Hinds bought of Henry Day & Son 8 bbl. flour @ \$6.50, 17 tons of hay @ \$8.75, 15 bbl. salt @ \$1.85, and 187 lb. sugar @ $9\frac{1}{4}$ cents.

QUESTIONS FOR REVIEW.

230. What is United States money? What is the table of United States money? What are coins? What is paper money? What are the gold coins of the United States? The silver coins? The nickel coins? The bronze coins?

How are cents changed to mills? Mills to cents? Dollars to cents? Cents to dollars? Dollars to mills? Mills to dollars?

What is a debt? What is a credit? Who is a debtor? Who is a creditor? What is an account? What is the balance of an account? What is a bill? What is the footing of a bill? When is a bill receipted? What are some of the abbreviations in common use?

What is the rule for multiplication of decimals? What is the rule for division of decimals? When, in division of decimals, the number of decimal places in the divisor exceeds the number in the dividend, what is done before dividing?

DENOMINATE NUMBERS

231. A Concrete Number is a number used in connection with some specified thing.

Thus, 5 books, 3 horses, 5 yards, are concrete numbers.

232. A Denominate Number is a concrete number in which the unit of measure is established by law or custom.

Thus, 3 pounds, 2 feet, 4 inches, are denominate numbers.

233. A Simple Denominate Number is a denominate number composed of units of the same denomination.

Thus, 5 feet, 9 pounds, 4 gallons, are simple denominate numbers.

234. A Compound Denominate Number is a denominate number composed of units of two or more denominations which are related to each other.

Thus, 6 feet, 4 inches; 3 pounds, 8 ounces; are compound denominate numbers.

MEASURES OF LENGTH.

235. Measures of Length, or Linear Measures, are used in measuring length and distances.

TABLE.

12 Inches (in.)	=	1 Foot	. .	ft.
3 Feet	=	1 Yard	. .	yd.
5½ Yards	}	= 1 Rod	. .	rd.
16½ Feet				
320 Rods	=	1 Mile	. .	mi.

EXERCISES.

1. How many in. are there in 2 ft. ? In 3 ft. ? In 4 ft. ?
2. How many ft. are there in 4 yd. ? In 5 yd. ? In 10 yd. ?
3. How many yd. are there in 4 rd. ? In 8 rd. ? In 10 rd. ?
4. How many ft. are there in 2 rd. ? In 4 rd. ? In 10 rd. ?
5. How many rd. are there in 2 mi. ? In 3 mi. ? In 8 mi. ?
6. How many ft. are there in 24 in. ? In 36 in. ? In 60 in. ?
7. How many yd. are there in 12 ft. ? In 18 ft. ? In 24 ft. ?
8. How many rd. are there in 33 ft. ? In 66 ft. ? In 99 ft. ?
9. How many mi. are there in 640 rd. ? In 906 rd. ?
10. How many in. are there in $\frac{1}{2}$ ft. ? In $\frac{2}{3}$ ft. ? In $\frac{3}{4}$ ft. ?
11. How many ft. are there in $\frac{1}{3}$ yd. ? In $2\frac{1}{3}$ yd. ? In $3\frac{2}{3}$ yd. ?
12. How many in. are there in 1 yd. ? In 1 yd. 1 ft. ?

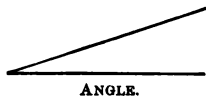
MEASURES OF SURFACE.

236. Measures of Surface, or Square Measures, are used in measuring any thing that has only length and breadth.

237. A Surface is any thing that has only length and breadth.

Thus, the floor, this page, or the outside of any thing is a surface.

238. An Angle is the difference in the direction of two lines that meet.



239. A Square is a figure that has four equal sides, and four equal corners or angles.



A *square inch* is a square whose sides are each one inch in length ; a *square foot* a square whose sides are each one foot in length.

SQUARE.

TABLE.

144 Square Inches (sq. in.)	= 1 Square Foot . sq. ft.
9 Square Feet	= 1 Square Yard . sq. yd.
$30\frac{1}{4}$ Square Yards	= 1 Square Rod . sq. rd.
160 Square Rods	= 1 Acre A.
640 Acres	= 1 Square Mile . sq. mi.

EXERCISES.

1. How many sq. in. are there in 2 sq. ft.? In 3 sq. ft.?
2. How many sq. ft. are there in 4 sq. yd.? In 5 sq. yd.?
3. How many sq. rd. are there in 2 A.? In 3 A.? In 4 A.?
4. How many sq. yd. are there in 27 sq. ft.? In 36 sq. ft.?
5. How many sq. ft. are there in 288 sq. in.? In 432 sq. in.?

MEASURES OF VOLUME.

240. Measures of Volume, or Cubic Measures, are used in measuring any thing that has length, breadth, and thickness.

241. A Solid is any thing that has length, breadth, and thickness.

Thus, a book, an orange, a block, are solids.

242. The Volume, or Solid Contents. of any thing is the number of solid units it contains.

243. A Cube is a solid having six equal square sides called faces.

A *cubic inch* is a solid whose sides or faces are each a square inch; a *cubic foot* is a solid whose sides or faces are each a square foot.



TABLE.

1728 Cubic Inches (cu. in.)	= 1 Cubic Foot . cu. ft.
27 Cubic Feet	= 1 Cubic Yard . cu. yd.
128 Cubic Feet	= 1 Cord Wood or Stone, C.

EXERCISES.

1. How many cu. in. are there in 2 cu. ft.? In 3 cu. ft.? In 4 cu. ft.? In 5 cu. ft.?
2. How many cu. ft. are there in 4 cu. yd.? In 6 cu. yd.? In 8 cu. yd.? In 10 cu. yd.?
3. How many cu. ft. are there in 5 C.? In 6 C.? In 10 C.? In 20 C.? In 30 C.?
4. How many cu. yd. are there in 54 cu. ft.? In 81 cu. ft.? In 108 cu. ft.?
5. How many cu. in. are there in $\frac{1}{2}$ cu. ft.? In $\frac{1}{4}$ cu. ft.?
6. How many cu. ft. are there in $\frac{1}{4}$ C.? In $\frac{1}{8}$ C.? In $\frac{1}{16}$ C.?

LIQUID MEASURE.

244. Liquid Measure is used in measuring liquids.

TABLE.

4 Gills (gi.)	= 1 Pint . . pt.
2 Pints	= 1 Quart . . qt.
4 Quarts	= 1 Gallon . . gal.

1. In determining the capacity of cisterns, reservoirs, etc., $31\frac{1}{2}$ gallons are considered a barrel (bbl.), and 2 barrels, or 63 gallons a hogshead (hhd.)
2. A gallon contains 231 cubic inches.

EXERCISES.

1. How many gi. are there in 2 pt.? In 5 pt.? In 8 pt.?
2. How many pt. are there in 5 qt.? In 8 qt.? In 12 qt.?
3. How many gi. are there in 2 qt.? In 5 qt.? In 10 qt.?
4. How many qt. are there in 5 gal.? In 6 gal.? In 10 gal.?
5. How many pt. are there in 16 gi.? In 20 gi.? In 36 gi.?
6. How many qt. are there in 20 pt.? In 40 pt.? In 30 pt.?
7. How many gal. are there in 40 qt.? In 36 qt.? In 38 qt.?
8. How many pt. are there in 8 gal.? In 12 gal.? In 18 gal.?

DRY MEASURE.

245. Dry Measure is used in measuring grain, roots, fruits, etc.

TABLE.

2 Pints (pt.)	=	1 Quart	. .	qt.
8 Quarts	=	1 Peck	. .	pk.
4 Pecks	=	1 Bushel	. .	bu.

A bushel contains 2150.4 cubic inches.

EXERCISES.

1. How many pints are there in 4 qt.? In 8 qt.?
2. How many quarts are there in 2 pk.? In 4 pk.?
3. How many pecks are there in 5 bu.? In 6 bu.?
4. How many bushels are there in 24 pk.? In 32 pk.?
5. How many pecks are there in 24 qt.? In 64 qt.?
6. How many quarts are there in 12 pt.? In 30 pt.?
7. How many pecks are there in $\frac{1}{2}$ bu.? In $\frac{1}{4}$ bu.?
8. How many bushels are there in 64 qt.? In 96 qt.?

AVOIRDUPOIS WEIGHT.

246. Avoirdupois Weight is used in weighing all coarse and heavy articles, as hay, grain, groceries, coal, including all metals, except *gold* and *silver*.

TABLE.

16 Ounces (oz.)	= 1 Pound lb.
100 Pounds	= 1 Hundred-weight . cwt.
20 Hundred-weight } 2000 Pounds	= 1 Ton T.

In weighing coal at the mines, and in levying duties, the long ton of 2240 lb. is sometimes used.

EXERCISES.

1. How many ounces are there in 2 lb.? In 3 lb.?
2. How many pounds are there in 5 cwt.? In 6 cwt.?
3. How many pounds are there in 2 T.? In 4 T.?
4. How many hundred-weight are there in 300 lb.?
5. How many tons are there in 2500 lb.? In 3000 lb.?
6. How many pounds are there in $\frac{1}{2}$ T.? In $\frac{1}{4}$ T.?

TROY WEIGHT.

247. Troy Weight is used in weighing gold, silver and jewels.

TABLE.

24 Grains (gr.)	= 1 Pennyweight . . pwt.
20 Pennyweights	= 1 Ounce oz.
12 Ounces	= 1 Pound lb.

The Troy lb. contains 5760 grains; the Avoirdupois lb., 7000 grains.

EXERCISES.

1. How many grains are there in 2 pwt.? In 3 pwt.? In 6 pwt.? In 8 pwt.?
2. How many pennyweights are there in 3 oz.? In 5 oz.?
3. How many ounces are there in 3 lb.? In 4 lb.?
4. How many grains are there in 1 oz.? In 2 oz.?
5. How many pounds are there in 24 oz.? In 36 oz.?
6. How many ounces are there in 40 pwt.? In 60 pwt.?
7. How many pennyweights are there in 48 gr.?

APOTHECARIES' WEIGHT.

248. Apothecaries' Weight is sometimes used by apothecaries and physicians in weighing medicines.

TABLE.

20 Grains (gr.)	= 1 Scruple	. . .	sc., or ℥.
3 Scruples	= 1 Dram	. . .	dr., or ℥.
8 Drams	= 1 Ounce	. . .	oz., or ℥.
12 Ounces	= 1 Pound	. . .	lb., or ℔.

EXERCISES.

1. How many grains are there in 3 sc.? In 4 sc.?
2. How many scruples are there in 3 dr.? In 5 dr.?
3. How many drams are there in 4 oz.? In 6 oz.?
4. How many ounces are there in 3 lb.? In 4 lb.?
5. How many pounds are there in 48 oz.? In 60 oz.?
6. How many ounces are there in 24 dr.? In 32 dr.?
7. How many drams are there in 9 sc.? In 15 sc.?
8. How many scruples are there in 40 gr.? In 60 gr.?

DIVISIONS OF TIME.

249. The ordinary divisions of time are given in the following

TABLE.

60 Seconds (sec.)	= 1 Minute . . .	min.
60 Minutes	= 1 Hour . . .	hr.
24 Hours	= 1 Day . . .	da.
7 Days	= 1 Week . . .	wk.
365 Days	= 1 Year . . .	yr.
366 Days	= 1 Leap Year . .	yr.
100 Years	= 1 Century . . .	C.

Centennial years whose number is exactly divisible by 400, and other years whose number is exactly divisible by 4, are *Leap Years*.

The months, their names and the number of days in each, are as follows:

January, 31 da. . .	Jan.	July, 31 da. . .	July.
February, 28 or 29 da.	Feb.	August, 31 da. . .	Aug.
March, 31 da. . .	Mar.	September, 30 da. . .	Sept.
April, 30 da. . .	Apr.	October, 31 da. . .	Oct.
May, 31 da. . .	May.	November, 30 da. . .	Nov.
June, 30 da. . .	June.	December, 31 da. . .	Dec.

EXERCISES.

1. How many seconds are there in 5 min.? In 6 min.?
2. How many minutes are there in 2 hr.? In 3 hr.?
3. How many minutes are there in 120 sec.? In 240 sec.?
4. How many hours are there in 2 da.? In 3 da.?
5. How many days are there in 4 wk.? In 6 wk.?
6. How many hours are there in 240 min.? In 180 min.?
7. How many weeks are there in 21 da.? In 36 da.?

COUNTING.

250. The following denominations are used in counting some classes of articles :

TABLE.

12 Things	=	1 Dozen	. . .	doz.
12 Dozen	=	1 Gross	. . .	gr.
12 Gross	=	1 Great Gross	G.	gr.

Two things are sometimes called a *pair*; six things, a *set*; and twenty things, a *score*.

STATIONERS' TABLE.

251. The following denominations are used in the paper trade:

TABLE.

24 Sheets	=	1 Quire.
20 Quires	=	1 Ream.
2 Reams	=	1 Bundle.

EXERCISES.

1. How many things are there in 2 doz.? In 4 doz.? In 6 doz.?
2. How many dozen are there in 3 gr.? In 4 gr.? In 2 G. gr.?
3. How many sheets are there in 3 quires? In 5 quires? In 10 quires?
4. How many quires are there in 3 reams? In 6 reams?

5. How many quires are there in 48 sheets? In 72 sheets? In 96 sheets?

6. How many reams are there in 40 quires? In 60 quires? In 80 quires?

7. How many years are 3 score years? 3 score and 10 years? 4 score years?

QUESTIONS FOR REVIEW.

252. What is a concrete number? What is a denominate number? What is a simple denominate number? What is a compound denominate number?

For what are linear measures used? Repeat the table of linear measure.

For what are square measures used? What is a surface? What is an angle? What is a square? What is a square inch? What is a square foot? What is the table of square measure?

For what are cubic measures used? What is a solid? What is meant by volume or solid contents? What is a cube? What is a cubic inch? What is a cubic foot? What is the table of cubic measure?

For what is liquid measure used? What is the table? In measuring cisterns, how many gallons are considered a barrel? How many gallons are considered a hogshead? How many cubic inches are there in a gallon?

For what is dry measure used? What is the table? How many cubic inches are there in a bushel?

For what is avoirdupois weight used? What is the table? Where is the *long* ton used? How many pounds are there in the *long* ton? For what is troy weight used? What is the table? How many grains are there in a troy pound? How many in a pound avoirdupois? For what is apothecaries' weight used? What is the table?

What are the ordinary divisions of time? What is the table? What years are leap years? Name the months, and the number of days in each. How does February sometimes have 29 days?

What is the table of counting? Repeat the stationers' table.

REDUCTION OF DENOMINATE NUMBERS.

253. 1. How many gills are there in 1 pt.? In 2 qt.? In 2 qt. 1 pt.?

2. How many feet are there in 1 yd.? In 2 yd.? In 2 yd. 2 ft.?

3. How many pecks are there in 1 bu.? In 3 bu.? In 3 bu. 3 pk.?

4. How many hours and minutes are there in 90 min.?

5. How many bushels and pecks are there in 12 pk.?

6. How many yards and feet are there in 20 ft.?

254. Reduction of a denominate number is the process of changing it from one denomination to another without altering its value.

255. Reduction Descending is the process of changing a denominate number to an equivalent number of a *lower* denomination.

256. Reduction Ascending is the process of changing a denominate number to an equivalent number of a *higher* denomination.

REDUCTION DESCENDING.

257. 1. How many inches are there in 6 yd., 2 ft., 5 in.?

PROCESS.

6 yd., 2 ft., 5 in.

3

20

12

245

ANALYSIS.—Since in 1 yd. there are 3 ft.,

in 6 yd. there are 6 times 3 ft., or 18 ft.; and 18 ft. + 2 ft. = 20 ft.

Since in 1 ft. there are 12 in., in 20 ft. there are 20 times 12 in., or 240 in.; 240 in. + 5 in. = 245 in.

Hence, in 6 yd., 2 ft., 5 in. there are 245 inches.

RULE.—*Multiply the number of the highest denomination given, by the number of units of the next lower denomination which is equal to one of the next higher, and to the product add the number given of this lower denomination.*

Proceed in like manner with this and each successive result thus obtained, until the number is reduced to the denomination required.

WRITTEN EXERCISES.

258. 2. How many quarts are there in 3 bu., 5 pk., 4 qt.?
3. How many inches are there in 5 yd., 2 ft., 4 in.?
4. How many pints are there in 6 gal., 3 qt., 1 pt.?
5. How many sec. are there in 2 hr., 20 min., 35 sec.?
6. How many oz. are there in 1 T., 3 cwt., 42 lb., 6 oz.?
7. How many hours are there in 1 wk., 5 da., 6 hr.?
8. How many grains are there in 5 lb., 6 oz., 8 pwt.?
9. Reduce 3 bu., 2 pk., 5 qt., 1 pt. to pints.
10. Reduce 2 rd., 3 yd., 2 ft., 8 in. to inches.
11. Reduce 9 sq. yd., 3 sq. ft., 15 sq. in. to square inches.
12. Reduce 2 A., 30 sq. rd., 5 sq. yd. to square yards.
13. Reduce 2 cu. yd., 8 cu. ft., 192 cu. in. to cubic inches.
14. Reduce 2 T., 3 cwt., 18 lb., 6 oz. to ounces.
15. Reduce 4 lb., 8 oz., 5 pwt., 10 gr. to grains.
16. Reduce 1 lb., 4 oz., 3 dr., 2 sc. to grains.
17. Reduce 3 reams, 10 quires, 12 sheets to sheets.
18. Reduce $\frac{3}{4}$ of a yard to inches.
19. Reduce $\frac{5}{7}$ of a week to minutes.
20. Reduce 4 gal., 3 qt., 1 pt. to gills.
21. Reduce 3 wk., 5 da., 7 hr. to hours.
22. Reduce 4 rd., 4 yd., 2 ft., 7 in. to inches.
23. Reduce 4 bu., 3 pk., 5 qt. to quarts.
24. Reduce $\frac{2}{3}$ of a bushel to quarts.
25. Reduce $\frac{3}{8}$ of a day to hours, minutes, etc.

REDUCTION ASCENDING.

259. 1. Reduce 2456 inches to units of higher denominations.

<p>PROCESS.</p> $ \begin{array}{r} 12 \overline{) 2456} \\ \underline{3) 204} \dots 8 \\ 5\frac{1}{2} \overline{) 68} \\ \underline{12} \dots 2 \end{array} $	<p>ANALYSIS.—Since there are 12 in. in 1 ft., in 2456 in. there are as many feet as 12 in. are contained times in 2456 in. $24\frac{5}{6}$ in. = 204 ft., 8 in.</p> <p>Since there are 3 ft. in 1 yd., in 204 ft. there are as many yards as 3 ft. are contained times in 204 ft. $29\frac{1}{3}$ ft. = 68 yd.</p> <p>Since in 1 rd. there are $5\frac{1}{2}$ yd., there are as many rods in 68 yd. as $5\frac{1}{2}$ yd. are contained times in 68 yd. $\frac{68}{5\frac{1}{2}}$ yd. = 12 rd., 2 yd.</p> <p>Hence, 2456 in. are equal to 12 rd., 2 yd., 8 in.</p>
--	---

RULE.—*Divide the given number by the number of that denomination which is equal to a unit of the next higher denomination.*

Divide the quotient in like manner, and thus proceed until the required denomination is reached.

The last quotient and the several remainders will be the result sought.

2. Reduce 1368 pints to bushels, pecks, etc.
3. Reduce 1605 inches to units of higher denominations.
4. Reduce 2364 pints to gallons, quarts, etc.
5. Reduce 16966 sec. to units of higher denominations.
6. Reduce 32684 ounces to tons, hundred-weight, etc.
7. Reduce 1729 hours to units of higher denominations.
8. Reduce 7695 grains troy to units of higher denominations.
9. Reduce 6165 grains apothecaries' weight to higher denominations.
10. Reduce 3684 square inches to higher units.

11. Reduce 28796 inches to units of higher denominations.
12. Reduce 275369 square inches to higher units.
13. Reduce 3297 sheets to reams, etc.
14. Reduce 64213 gills to gallons, etc.
15. Reduce 12326 feet to miles, etc.

ADDITION OF COMPOUND NUMBERS.

260. 1. Find the sum of 5 yd., 2 ft., 8 in.; 6 yd., 1 ft., 11 in.; 4 yd., 2 ft., 9 in.; 3 yd., 1 ft.

PROCESS.			ANALYSIS.—The numbers are written so that units of the same denomination stand in the same column, and we begin at the right to add.
yd.	ft.	in.	
5	2	8	The sum of the inches is 28 in., which is equal to 2 ft., 4 in. The 4 is written under the inches and the 2 ft. is reserved to add with the feet.
6	1	11	
4	2	9	
3	1		The sum of the feet is 8 ft. which is equal to 2 yd., 2 ft. The 2 is written under the feet and the 2 yd. reserved to add with the yards.
20	2	4	

The sum of the yards is 20 yards, which is written under yd.

Therefore, the sum is 20 yd., 2 ft., 4 in.

RULE.—*Change the rule for addition of simple numbers so that it may be applicable to compound denominate numbers.*

2. What is the sum of 6 gal., 3 qt., 1 pt.; 5 gal., 2 qt., 1 pt.; 4 gal., 3 qt., 1 pt.; 6 gal., 1 qt., 1 pt.?

3. What is the sum of 12 lb., 4 oz., 8 pwt.; 9 lb., 6 oz., 17 pwt.; 8 lb., 10 oz., 10 pwt.?

4. What is the sum of 18 bu., 3 pk., 5 qt.; 12 bu., 2 pk., 7 qt.; 9 bu., 3 pk., 6 qt.; 8 bu., 1 pk., 5 qt.?

5. What is the sum of 7 da., 10 hr., 12 min., 35 sec.; 10 da., 15 hr., 45 min., 30 sec.; 26 da., 18 hr., 30 min., 50 sec.?

6. What is the sum of 8 A., 19 sq. rd.; 24 A., 64 sq. rd.; 31 A., 69 sq. rd.; 64 A., 48 sq. rd.?

7. What is the sum of 16 cu. yd., 18 cu. ft.; 24 cu. yd., 19 cu. ft.; 19 cu. yd., 23 cu. ft.; 26 cu. yd., 19 cu. ft.?

8. What is the sum of 3 T., 4 cwt., 18 lb., 6 oz.; 4 T., 12 cwt., 70 lb., 11 oz.; 5 T., 9 cwt. 60 lb., 10 oz.; 9 T., 8 cwt., 15 lb., 7 oz.?

9. What is the sum of 4 lb., 6 oz., 4 dr., 2 sc., 10 gr.; 5 lb., 8 oz., 5 dr., 2 sc., 12 gr.; 4 lb., 5 oz., 4 dr., 1 sc., 12 gr.?

SUBTRACTION OF COMPOUND NUMBERS.

261. 1. From 8 yd., 1 ft., 5 in. subtract 3 yd., 2 ft., 8 in.

PROCESS.			ANALYSIS. —The numbers are written so that units
yd.	ft.	in.	of the same order stand in the same column and we
8	1	5	begin at the right to subtract.
3	2	8	Since 8 in. can not be subtracted from 5 in., a unit
<hr/>			of the next higher order is united with the 5 in.,
4	1	9	making 1 ft. 5 in., or 17 in. Then 8 in. from 17 in.

leave 9 in. The 9 is written under inches in the remainder.

Since 1 ft. was united with 5 in. there are no feet remaining in the minuend. Inasmuch as 2 ft. can not be subtracted from 0 ft., a unit of the next higher order is united with 0 ft., making 3 ft. Then 2 ft. from 3 ft. leave 1 ft. The 1 is written under feet in the remainder.

Since 1 yd. was changed into feet there are but 7 yd. left in the minuend. 3 yd. from 7 yd. leave 4 yd.

Hence, the remainder is 4 yd., 1 ft., 9 in.

RULE.—*Change the rule for subtraction of simple numbers so that it may be applicable to compound denominate numbers.*

2. From 8 gal., 3 qt., 1 pt., 2 gi. subtract 3 gal., 3 qt., 1 pt., 3 gi.

3. From 12 lb., 4 oz., 6 pwt., 8 gr. subtract 5 lb., 9 oz., 4 pwt., 10 gr.

4. From 20 bu., 3 pk., 4 qt. subtract 11 bu., 3 pk., 7 qt.

5. From 9 da., 10 hr., 8 min., 26 sec., subtract 4 da., 6 hr., 9 min., 15 sec.
6. From 213 A., 111 sq. rd. subtract 110 A., 130 sq. rd.
7. From 18 C., 16 cu. ft. subtract 10 C., 26 cu. ft.
8. From 100 bu. subtract 29 bu., 3 pk., 7 qt., 1 pt.
9. From 6 T., 8 cwt., 46 lb., 10 oz. subtract 3 T., 7 cwt., 45 lb., 12 oz.
10. From 10 reams, 6 quires, 8 sheets subtract 5 reams, 8 quires, 16 sheets.
11. From 19 sq. yd., 8 sq. ft., 36 sq. in. subtract 15 sq. yd., 7 sq. ft., 44 sq. in.
12. From 8 rd., 4 yd., 2 ft., 9 in. subtract 4 rd., 2 yd., 3 ft., 11 in.
13. From the sum of 13 lb., 6 oz., 4 dr.; 8 lb., 7 oz., 8 dr., and 8 lb., 3 oz., 5 dr. subtract the sum of 4 lb., 10 oz., 5 dr.; 6 lb., 3 oz., 4 dr.; 2 lb., 9 oz., 7 dr.
14. From the sum of 2 T., 3 cwt., 18 lb., 6 oz.; 5 T., 13 cwt., 16 lb., 8 oz., and 3 T., 8 cwt., 15 oz. subtract the sum of 3 T., 19 cwt., 12 lb., 9 oz., and 2 T., 8 cwt., 17 lb., 10 oz.
15. How long was it from Jan. 10, 1841, to May 7, 1853?

PROCESS.			ANALYSIS.—Since the later date expresses the greater period of time, we write it as the minuend, and the earlier date as the subtrahend, giving the month its number instead of the name. We then subtract as in denominate numbers, considering 30 days one month, and 12 months one year. The remainder will be the time as correctly as it can be expressed in months and days.
1853	5	7	
1841	1	10	
<hr/>			
12	3	27	

16. How long was it from Feb. 10, 1810, to May 8, 1846?
17. How long was it from June 3, 1843, to Sept. 1, 1871?
18. How long was it from Dec. 9, 1865, to April 8, 1882?
19. How long was it from Feb. 28, 1843, to April 7, 1864?
20. How long was it from March 9, 1812, to Dec. 10, 1876?

MULTIPLICATION OF COMPOUND NUMBERS.

262. 1. How much is 4 times 4 yd., 1 ft., 10 in. ?

PROCESS. **ANALYSIS.**—The numbers are written as in simple numbers, and we begin at the right to multiply.

yd.	ft.	in.
4	1	10
		4
18	1	4

4 times 10 in. are 40 in., or 3 ft., 4 in. The 4 in. is written under inches in the product, and the 3 ft. reserved to add with the feet.

4 times 1 ft. are 4 ft.; 4 ft. + 3 ft. reserved are 7 ft., or 2 yd., 1 ft. The 1 ft. is written under feet in the product and the 2 yd reserved to add with the yards.

4 times 4 yd. are 16 yd.; 16 yd. + 2 yd. reserved are 18 yd.

Therefore, the product is 18 yd., 1 ft., 4 in.

RULE.—*Modify the rule for the multiplication of simple numbers so that it may be applicable to compound denominate numbers.*

2. Multiply 7 yd., 2 ft., 11 in. by 4.
3. Multiply 3 bu., 5 pk., 4 qt. by 7.
4. Multiply 6 gal., 3 qt., 1 pt. by 8.
5. Multiply 2 hr., 20 min., 35 sec. by 6.
6. Multiply 3 cwt., 42 lb., 6 oz. by 5.
7. Multiply 5 lb., 6 oz., 8 pwt. by 9.
8. Multiply 2 rd., 3 yd., 2 ft., 8 in. by 6.
9. Multiply 4 lb., 4 oz., 3 dr., 4 sc. by 7.
10. Multiply 12 cu. yd., 9 cu. ft. by 12.
11. Multiply 5 sq. yd., 6 sq. ft., 110 sq. in. by 12.
12. Multiply 3 reams, 10 quires, 12 sheets by 9.
13. Multiply 6 yd., 2 ft., 7 in. by 8.
14. Multiply 5 gal., 3 qt., 1 pt., 3 gi. by 7.
15. Multiply 7 lb., 6 oz., 5 pwt., 8 gr. by 9.
16. Multiply 3 rd., 4 yd., 1 ft., 7 in. by 12.
17. Multiply 3 da., 8 hr. 15 min., 10 sec. by 8.
18. Multiply 8 sq. yd., 7 sq. ft., 15 sq. in. by 10.

DIVISION OF COMPOUND NUMBERS.

263. 1. Divide 17 yd., 2 ft., 4 in. by 5.

PROCESS.			ANALYSIS.—The numbers are written as in simple numbers and we begin at the left to divide.
yd.	ft.	in.	
5) 17	2	4	One-fifth of 17 yd. is 3 yd. and 2 yd. remaining. The 3 yd. is written in the quotient and the 2 yd. remaining united with the feet, making 8 ft.
3	1	8	

One-fifth of 8 ft. is 1 ft. and 3 ft. remaining. The 1 ft. is written in the quotient and the 3 ft. remaining united with the inches, making 40 inches.

One-fifth of 40 inches is 8 in.

Therefore, the quotient is 3 yd., 1 ft., 8 in.

RULE.—*Change the rule for division of simple numbers so that it may be applicable to compound denominate numbers.*

2. Divide 12 yd., 2 ft., 7 in. by 6.
3. Divide 13 bu., 3 pk., 5 qt. by 4.
4. Divide 15 gal., 3 qt., 1 pt. by 4.
5. Divide 22 hr., 20 min., 35 sec. by 5.
6. Divide 17 cwt., 42 lb., 6 oz. by 6.
7. Divide 21 lb., 6 oz., 8 pwt. by 8.
8. Divide 14 lb., 4 oz., 3 dr., 2 sc. by 5.
9. Divide 22 cu. yd., 9 cu. ft. by 8.
10. Divide 36 sq. yd., 6 sq. ft., 110 sq. in. by 5.

MEASUREMENT.

264. To compute the area of rectangular surfaces.

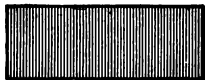
1. What is a square inch? How many square inches are there in a strip of paper 10 inches long and 1 inch wide?

2. How many square inches are there in a strip of paper 10 inches long and 2 inches wide?

3. How many square feet are there in a plat of ground 12 feet long and 1 foot wide? In a plat 12 feet long and 2 feet wide? In a plat 10 feet long and 3 feet wide?

4. How many square feet are there in a floor 12 ft. long and 4 ft. wide?

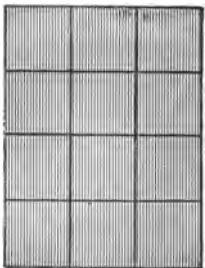
265. A **Rectangle** is a figure that has four straight sides and four equal angles.



RECTANGLE.

The angles of a rectangle are all right angles.

266. The **Area** or extent of any surface is the number of square units it contains.



Thus, if a rectangle is 4 inches long and 3 inches wide the area will be 12 square inches.

For it may be divided into 4 rows, each containing 3 square *inches* or *units*, and the entire area will be 12 square inches.

The method of computing the area of figures that are not rectangular is given in **MENSURATION** in *The Practical Arithmetic*.

WRITTEN EXERCISES.

267. 1. What is the area of a plat of ground 12 ft. long and 6 ft. 8 in. broad?

PROCESS. $12 \times 6\frac{2}{3} = 80$ sq. ft. **ANALYSIS.**—Since a plat 12 ft. long and 1 ft. broad contains 12 sq. ft., a plat 6 ft. 8 in., or $6\frac{2}{3}$ ft., broad contains $6\frac{2}{3}$ times 12 sq. ft., or 80 sq. ft.

RULE.—*Multiply together the numbers expressing the length and breadth.*

The length and breadth must be expressed in units of the same denomination.

2. A room is 15 feet wide and 18 feet long. What is the area of the floor?

3. What is the area of a floor that is 40 ft. wide and 10 ft. long?

4. How many square yards of carpet will be required to cover a floor 18 ft. wide and 21 ft. long?

5. How many square yards are there in the surface of a wall 24 feet long and 12 feet high?

6. How much will it cost to plaster a wall 15 ft. long and 12 ft. high at 27 cents per square yard?

7. How many yards of carpet 1 yard wide will be required to cover a room 18 ft. wide and 23 ft. 6 in. long?

8. How much will it cost to carpet a room 18 ft. long and 12 ft. wide if the carpet is 1 yd. wide and cost \$1.15 per yd.?

9. How many acres are there in a field that is 60 rods wide and 90 rods long?

10. How many acres are there in a field that is 80 rods wide and 100 rods long?

11. How much must be paid for mowing a field 30 rods wide and 64 rods long at \$1.25 per acre?

12. A pasture containing 10 acres was 20 rods wide. How long was it?

268. To compute the volume of rectangular bodies.

1. What is a cubic foot? How many cubic feet are there in a block of stone 5 feet long, 1 foot wide and 1 foot thick?

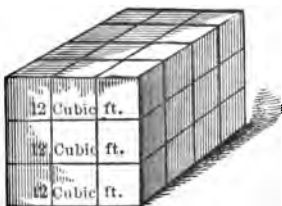
2. How many cubic feet are there in a block that is 5 feet long, 2 feet wide and 1 foot thick?

3. How many cubic feet are there in a block that is 5 feet long, 2 feet wide and 2 feet thick?

4. How many cubic feet are there in a block that is 5 feet long, 3 feet wide and 1 foot thick? In a block 5 feet long, 3 feet wide and 2 feet thick?

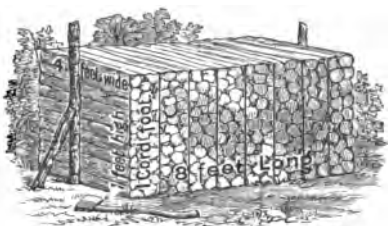
269. The **Volume, or Solid Contents**, of any body is the number of solid units it contains.

Thus, if a solid is 4 ft. long, 3 ft. wide and 3 ft. thick, its volume will be 36 cubic feet. For it may be divided into 3 blocks, each containing 12 cubic feet, making in all 36 cubic feet.



270. A **Cord** of wood or stone is a pile 8 feet long, 4 feet wide and 4 feet high.

A pile that is 1 foot long, 4 feet wide and 4 feet high is a cord foot.



A **Perch** of stone contains 24.75 cubic feet.

WRITTEN EXERCISES.

271. 1. How many cubic feet are there in a block of stone 6 ft. long, 4 ft. 6 in. wide and 2 ft. 4 in. thick?

PROCESS.

$$6 \times 4\frac{1}{2} \times 2\frac{1}{3} = 63 \text{ cu. ft.}$$

ANALYSIS.—Since in a block 6 ft.

long, 1 ft. wide and 1 ft. thick there are 6 cu. ft., in a block 6 ft. long, $4\frac{1}{2}$ ft. wide and 1 foot thick there are $4\frac{1}{2}$ times 6 cu. ft., or 27 cu. ft., and in a block 6 ft. long, $4\frac{1}{2}$ ft. wide and $2\frac{1}{3}$ ft. thick there are $2\frac{1}{3}$ times 27 cu. ft., or 63 cu. ft.

RULE.—*Multiply together the numbers expressing the length, breadth and thickness.*

The length, breadth and thickness must be expressed in units of the same denomination.

2. What are the contents of a rectangular solid 6 ft. long, 4 ft. wide and 3 ft. thick?

3. What are the contents of a rectangular solid 8 ft. long, 5 ft. wide and 4 feet thick?
4. How many cubic feet are there in a block of marble 10 ft. long, 5 ft. wide and 3 ft. 6 in. thick?
5. How many cubic feet are there in a cube whose edges are 6 ft. long?
6. How many cubic feet are there in 5 cords of stone?
7. How many cords of wood are there in a pile 24 ft. long, 8 ft. wide and 8 ft. high?
8. How many cords are there in a pile of wood 30 ft. long, 4 ft. wide and 6 ft. high?
9. If a range of wood is 6 ft. high and 8 ft. wide, how long must it be to contain 30 cords?
10. How many cubic yards of earth must be excavated to make a cellar 33 ft. long, 16 ft. wide and 7 ft. deep?
11. What will it cost to excavate a cellar 40 ft. long, 18 ft. wide and 8 ft. deep, at \$.35 per cubic yard?
12. How many perch of stone are there in a wall that is 28 ft. long, 1 ft. 6 in. thick and 5 ft. high?

272. To measure lumber by board measure.

273. A Board Foot is 1 ft. long, 1 ft. wide and 1 inch thick. Hence, when a board is 1 inch thick, the number of feet board measure may be found by the following rule:

RULE.—*Multiply the length in feet by the breadth expressed in feet. Or,*

Multiply the length in feet by the breadth in inches and divide the result by 12.

When lumber is *more than an inch* thick, the number of feet board measure may be found by the following rule:

RULE.—*Multiply the length in feet by the breadth in feet, and this product by the number of inches in thickness.*

WRITTEN EXERCISES.

274. 1. How many feet board measure are there in a board 18 ft. long, 9 in. wide, 1 in. thick?

2. What are the contents board measure of a joist 12 ft. long, 1 ft. wide, and 2 in. thick?

3. How many feet board measure are there in a board 15 ft. long, 10 in. wide, and 1 in. thick?

4. How many feet board measure are there in 5 boards, each 16 ft. long, 10 in. wide, 1 in. thick?

5. How many feet board measure are there in 5 joists, each 16 ft. long, 14 in. wide, and 3 in. thick?

6. Find the cost of 12 scantling 8 in. wide, 4 in. thick, and 16 ft. long at \$20 per thousand feet, or 2 cents per foot.

7. Find the cost of 30 boards 12 ft. long, 11 in. wide, and 1 in. thick at \$40 per thousand feet.

REVIEW EXERCISES.

ORAL EXERCISES.

275. 1. What will be the cost of 2 lb. 8 oz. of butter at 30 cents per pound?

2. How much must I pay for 3 pk. of chestnuts at 5 cents per quart?

3. How much will 5 yd. of wire cost at 4 cents per foot?

4. How much will 6 gal. milk cost at 3 cents a pint?

5. How many hours are there in 2 da., 2 hr.?

6. A man worked 2 hr., earning 5 cents per minute. How much did he earn?

7. If silver is worth \$1 per ounce, how much must I pay for 2 lb. 6 oz.?

8. How many packages of seed, each containing $\frac{1}{2}$ pt., can be made from 1 bu., 2 pk.?

9. A man jumped 2 rd. at 3 jumps. What was the average distance jumped each time?

10. A man having 2 sq. yd. of silk cut it into strips containing 3 sq. ft. each. How many strips were there?

11. A stationer sold 2 quires, 6 sheets of paper at 2 cents per sheet. How much did he get for it?

12. A lad picked 48 qt. of berries, which he sold at \$1 per bushel. How much did he get for them?

13. A merchant sold in one year 480 ounces of indigo. How much did he get for it if the selling price was \$2 per pound?

14. How many blocks, each a cubic inch, can be formed from a block which is a cubic foot?

15. A rope 12 yd., 2 ft. long was cut into 2 equal parts. How long was each part?

16. How much will 33 eggs cost at 12 cents per dozen?

17. From a keg of molasses containing 5 gal., 3 gal., 2 qt. were drawn out. How much remained?

18. How many weeks does a boy spend at his grandfather's, if he remains there 49 days?

19. A man purchased at one time 2 bu., 3 pk. of clover seed, and at another 3 bu., 7 pk. How much did he buy?

20. A man traveling by carriage found that he had just 3 hours in which to reach the railway station. He reached it in 2 h., 35 min. How much time had he to spare?

21. A merchant had 4 remnants each containing 3 yd., 2 ft. How much was there in all?

22. Five boys together gathered 2 bu., 6 qt. of chestnuts. How many quarts should each receive for his share?

23. A stationer bought paper at \$2.55 per ream and sold it at 20 cents per quire. How much did he gain?

24. A cabinet-maker bought a plank of black-walnut 12 feet long, 1 foot wide, and 2 inches thick. If he paid 10 cents per ft. board measure for it, how much did it cost him?

WRITTEN EXERCISES.

276. 1. From a piece of cloth containing 13 yd., 1 ft. a tailor cut off 5 yd., 2 ft. How much was left?

2. Bought 15 bu. of oats at $37\frac{1}{2}$ cents per bushel, and sold them at 15 cents a peck, How much did I gain?

3. What will be the cost of 13 lb., 8 oz. of butter at 30 cents per pound?

4. What will be the cost of 4200 lb. of hay at \$1.80 per hundred-weight?

5. A printer used 5 reams, 8 quires, 15 sheets of paper. How many sheets did he use?

6. An apothecary made 1 oz. of quinine into two-grain pills. How many pills did he make?

7. A farmer sold a range of wood 16 ft. long, 4 ft. wide, and 6 ft. high, at \$3.50 per cord. How much did he receive for it?

8. A man's crop from 5 acres was 247 bu., 3 pk., 7 qt. of wheat. What was the average yield per acre?

9. How many minutes are there in a common year of 365 days?

10. A wagon wheel is 15 ft., 6 in. around. How many revolutions will it make in going 10 miles?

11. If a bushel of wheat weighs 60 lb., how many pounds will 36 bu., 3 pk. weigh?

12. How much will it cost to carpet a room 24 ft. long and 18 ft. wide at \$1.20 per square yard?

13. What will it cost to pave a rectangular court 80 ft. long and 50 ft. wide at 50 cents per square yard?

14. What will it cost to pave a street 50 feet wide and 1500 feet long with Nicholson pavement at 30 cents per square foot?

15. The contents of 4 barrels were as follows: 33 gal., 3

qt., 2 pt.; 42 gal., 2 qt., 1 pt.; 37 gal., 1 qt., 1 pt.; 38 gal., 1 pt. What were the contents of all?

16. If a ship sails 9 miles per hour, how long will it take her to sail 1747 miles?

17. A milkman sold 32 gal., 3 qt., 1 pt. of milk daily. How much did he sell in 12 days?

18. A man sold 3 T., 17 cwt. of hay at \$1.80 per cwt. How much did he receive for it?

19. How many feet board measure are there in 6 joists each 15 ft. long, 14 in. wide, and 3 in. thick?

20. What will be the expense of painting a roof 48 feet long and 22 feet wide at \$.30 per square yard?

21. A milkman sold 684 pints of milk at 20 cents per gallon. How much did he receive for it?

22. A jeweler having 36 lb., 10 oz., 14 pwt. of silver made 8 teapots from it, and had left 21 lb., 6 oz. What was the weight of each teapot?

23. What will be the cost of 5 gal., 3 qt., 1 pt. of alcohol at \$.30 per quart?

24. How many cubic feet are there in a block of marble which is 6 ft. long, 4 ft. wide, and 3 ft. thick?

25. A silversmith sold 5 lb., 2 oz. of silver plate for \$93. How much did he get for it per ounce?

26. A man divided a farm of 215 A., 135 sq. rd. equally among 3 sons. What was each son's share?

27. How many yards of matting $1\frac{1}{4}$ yards wide will be required to cover a floor that is 20 ft. long and 15 ft. wide?

28. The American Revolution began April 19, 1775, and ended 8 yr., 6 mo., 14 da. afterward. When was the treaty signed by which it was ended?

29. Which is heavier, and how much in grains, a pound of gold or a pound of iron?

30. How many cannon balls each weighing 66 lb., $10\frac{1}{2}$ oz. can be made from 1 ton of iron?

31. What will be the cost of 20 planks 18 ft. long, 8 in. wide, and 2 in. thick, at \$25. per thousand feet?

32. A cask of wine contained 31 gal., 3 qt. What was it worth at \$2.50 per gallon?

33. How long will it require for a person to count \$100,000, if he counts one dollar per second for 10 hours per day?

34. How many pieces of tape, each containing 6 yd., 2 ft., 3 in., can be made from 320 yd.?

35. How wide must a piece of land be to contain two acres, if it is 25 rods in length?

36. How many yards of plastering are there on the sides of a cistern 12 ft. long, 8 ft. wide, and 6 ft. deep?

37. If a railroad train travels at the rate of 32 miles per hour, how many miles will it travel in 2 da., 8 hr., 30 min.?

QUESTIONS FOR REVIEW.

277. What is reduction of denominate numbers? What is reduction descending? Reduction ascending? What is the rule for reduction descending? For reduction ascending?

What is the rule for the addition of compound numbers? For subtraction of compound numbers? How are dates subtracted? What is the rule for multiplication of compound numbers? What is the rule for division of compound numbers?

What is a rectangle? What is the area of a surface? How is the area of rectangular surfaces computed? What is a volume or solid? What is a cord of wood? What is a perch of stone? How is the volume of rectangular bodies computed? What is a board foot? How is the number of board feet computed when the board is 1 inch thick? How when it is more than 1 inch thick?

What is a concrete number? What is a denominate number? A simple denominate number? A compound denominate number?

What is an angle? What is a square? What are right angles? What is a surface? What is a cube? What is a square foot? A cubic foot?

PERCENTAGE

278. 1. In a regiment of soldiers 2 out of 50 were killed. How many was that *per hundred*, or *per cent.*?

2. In a school 3 out of 20 pupils are more than 14 years of age. How many is that *per hundred*, or *per cent.*?

3. How much is $\frac{1}{100}$, or 1 *per cent.*, of \$200? Of \$400? Of \$800?

4. How much is $\frac{3}{100}$, or 3 *per cent.*, of \$500? Of \$700? Of \$900?

5. How much is $\frac{5}{100}$, or 5 *per cent.*, of \$600? Of \$700? Of \$400?

6. How many hundredths of any thing is $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{1}{8}$ of it?

7. How many hundredths of any thing is $\frac{1}{8}$ of it? $\frac{1}{10}$ of it? $\frac{1}{20}$ of it?

279. *Per cent.* means *by the hundred*.

Thus 3 *per cent.* means 3 out of every 100, or 3 hundredths.

280. The **Sign of Per cent.** is %.

Thus 8% is read 8 *per cent.*

281. **Percentage** treats of computations which involve *per cent.*

282. Since *per cent.* is a number of hundredths it may be expressed by a decimal or by a common fraction.

2 per cent. is written	2%, .02, or $\frac{2}{100}$.
5 per cent. is written	5%, .05, or $\frac{5}{100}$.
47 per cent. is written	47%, .47, or $\frac{47}{100}$.
135 per cent. is written	135%, 1.35, or $\frac{135}{100}$.
12½ per cent. is written	12½%, .12½, or $\frac{12\frac{1}{2}}{100}$.
$\frac{3}{4}$ per cent. is written	$\frac{3}{4}$ %, .00¾, or $\frac{3}{400}$.
31¼ per cent. is written	31¼%, .31¼, or $\frac{31\frac{1}{4}}{100}$.

EXERCISES.

283. Express decimally the following:

1. 10%.	6. 75%.	11. 12½%.
2. 15%.	7. 90%.	12. 37½%.
3. 37%.	8. 85%.	13. 87½%.
4. 25%.	9. 69%.	14. 66⅔%.
5. 20%.	10. 125%.	15. $\frac{3}{4}$ %.

Express as common fractions in their smallest terms:

16. 10%.	20. 12½%.	24. 80%.
17. 15%.	21. 16⅔%.	25. 40%.
18. 25%.	22. 75%.	26. 125%.
19. 20%.	23. 50%.	27. 150%.

Express in hundredths, or as per cent.:

28. $\frac{1}{2}$ of a number.	38. $\frac{3}{8}$ of a number.
29. $\frac{1}{4}$ " " "	39. $\frac{5}{16}$ " " "
30. $\frac{1}{8}$ " " "	40. $\frac{3}{40}$ " " "
31. $\frac{1}{10}$ " " "	41. $\frac{7}{20}$ " " "
32. $\frac{1}{20}$ " " "	42. $\frac{4}{25}$ " " "
33. $\frac{1}{25}$ " " "	43. $\frac{12}{25}$ " " "
34. $\frac{3}{4}$ " " "	44. $\frac{3}{200}$ " " "
35. $\frac{3}{10}$ " " "	45. $\frac{8}{500}$ " " "
36. $\frac{4}{25}$ " " "	46. $\frac{2}{700}$ " " "
37. $\frac{1}{8}$ " " "	47. $\frac{5}{9000}$ " " "

284. To find the Percentage when the Base and Rate are given.

1. What is 10 per cent., or $\frac{10}{100}$, of \$50? Of \$60?
2. What is 5 per cent., or $\frac{5}{100}$, of 500 bu.? Of 600 bu.?
3. What is 25 per cent., or $\frac{25}{100}$, of 200 gal.? Of 300 gal.?
4. What is 40 per cent. of 200 tons? Of 400 tons?
5. What is $33\frac{1}{3}$ per cent. of 300 horses? Of 600 horses?

285. The Base is the number, of which the per cent. is taken.

286. The Rate is the number of hundredths taken.

287. The Percentage is the number, which is a certain number of hundredths of the base.

WRITTEN EXERCISES.

288. 1. What is 5 per cent. of \$24.36?

PROCESS.

$$\$24.36 \times \frac{1}{20} = \$1.218$$

Or,

$$\$24.36 \times .05 = \$1.218$$

FORMULA.

$$B \times R = P.$$

ANALYSIS.—Since 5% of any number is $\frac{1}{20}$, or $\frac{1}{20}$ of it, 5% of \$24.36 is $\frac{1}{20}$ of \$24.36, which is \$1.218. Or,

Since 5% of any number is .05 of it, 5% of \$24.36 is .05 of \$24.36, which is \$1.218.

RULE.—Multiply the base by the rate.

2. Find 10% of \$18.69.
3. Find 15% of \$16.40.
4. Find 35% of \$20.36.
5. Find $12\frac{1}{2}\%$ of 848 gal.
6. Find $16\frac{2}{3}\%$ of 672 mi.
7. Find $37\frac{1}{2}\%$ of 432 in.
8. A farmer lost 25% of a flock of 440 sheep. How many did he lose?
9. A merchant sold 35% of a quantity of goods worth \$24000. How many dollars' worth did he sell?

10. A man purchased a farm for \$13000 and sold it at a gain of 20%. How much did he receive for it?

11. A man invested $12\frac{1}{2}\%$ of \$16240 in a silver mine. How much did he invest?

12. A man spent annually 83% of his salary, which was \$2000. How much did he spend?

289. To find the Rate when the Percentage and Base are given.

1. If a man earns \$40 and spends \$20 of it, what part of it does he spend? How many hundredths of it? How many per cent.?

2. If I spend $\frac{1}{4}$ of my money, what per cent. of it do I spend?

3. A man had 40 horses, but he sold 10 of them. What per cent. of them did he sell?

4. What part of \$15 is \$3? What per cent.?

WRITTEN EXERCISES.

290. 1. What per cent. of 375 yards are 150 yards?

PROCESS.

$$150 \text{ yd.} = \frac{150}{375} \text{ of } 375 \text{ yd.} =$$

$$\frac{2}{3} \text{ of } 375 \text{ yd.} = \frac{40}{100}, \text{ or}$$

$$40\% \text{ of } 375 \text{ yd.,}$$

Or,

$$150 \text{ yd.} \div 375 \text{ yd.} = .40 = 40\%$$

FORMULA.

$$P \div B = R.$$

ANALYSIS.—150 yd.

are $\frac{150}{375}$, or $\frac{2}{3}$ of 375 yd.

$\frac{2}{3}$ expressed as hundredths is equal to 40 hundredths; therefore, 150 yd. are .40, or 40% of 375 yd. Or,

Since the percentage is the product of

the base by the rate, if the percentage is divided by the base the result is the rate. Therefore, $150 \div 375 = .40$, or 40%.

RULE.—*Divide the percentage by the base.*

What per cent. of:

2. 200 acres are 50 acres?
3. 800 hours are 160 hours?
4. 960 gal. are 320 gal.?
5. 600 bu. are 450 bu.?

What per cent. of:

6. \$320 are \$200?
7. \$425 are \$225?
8. \$340 are \$85?
9. \$432 are \$72?

10. A farmer sold 210 acres from a farm containing 540 acres? What per cent. of the farm did he sell?

11. A grocer sold tea for \$1 per pound that cost him \$.75. What per cent. of the cost did he gain?

12. If I sell butter for 42 cents per pound that cost me 35 cents, what per cent. of the cost do I gain?

13. A man earned \$160 per month and spent \$120. What per cent. of his earnings did he spend?

14. A man who worked 45 days on a farm spent 9 days of the time in plowing. What per cent. of his time did he plow?

15. A farmer who raised 600 bushels of grain sold 450 bushels of it. What per cent. of his grain did he sell?

16. The receipts at a concert were \$450 and the expenses \$360. What per cent. of the receipts was gained?

291. To find the Base when the Percentage and Rate are given.

1. A man spent \$20 which was 10 per cent., or $\frac{10}{100}$, of all the money he had. How much money had he?

2. Of what sum is \$18 50%, or $\frac{50}{100}$, or $\frac{1}{2}$?

3. Of what sum is \$20 40%, or $\frac{40}{100}$, or $\frac{2}{5}$?

4. Of what sum is \$30 60%, or $\frac{60}{100}$, or $\frac{3}{5}$?

5. Of what sum is \$50 25%, or $\frac{25}{100}$, or $\frac{1}{4}$?

6. Of what sum is \$40 10 per cent.?

7. Of what sum is \$30 15 per cent.?

8. Of what sum is \$60 20 per cent.?

9. Of what sum is \$160 80 per cent.?

WRITTEN EXERCISES.

292. 1. A merchant sold 75% of a quantity of silk for \$300. What would he receive for the whole at the same rate?

PROCESS.

$$75\% \text{ or } \frac{3}{4} = \$300$$

$$\frac{1}{4} = \$100$$

$$\text{Whole } \$400$$

Or,

$$\$300 \div .75 = \$400$$

FORMULA.

$$P \div R = B.$$

ANALYSIS.—Since 75%, or $\frac{3}{4}$, or $\frac{75}{100}$, of the value of the goods was \$300, $\frac{1}{4}$ of the value was $\frac{1}{4}$ of \$300, or \$100; and since \$100 was $\frac{1}{4}$ of the value, the value was 4 times \$100, or \$400. Or,

Since the percentage is the product of the base multiplied by the rate, if the percentage is divided by the rate, the quotient will be the base. Therefore, $\$300 \div .75 = \400 , the entire value.

RULE.—*Divide the percentage by the rate.*

Of what number is:

2. $312\ 25\%$?

3. $250\ 40\%$?

4. $420\ 33\frac{1}{3}\%$?

5. $650\ 20\%$?

Of what number is:

6. $\$216.15\ 62\frac{1}{2}\%$?

7. $23.5\ \text{bu.}\ 16\frac{2}{3}\%$?

8. $225\ \text{men}\ 15\%$?

9. $363\ \text{oxen}\ 37\frac{1}{2}\%$?

10. A clerk spent \$800 per year, which was 80% of his entire salary. How much was his salary?

11. A farmer sold 320 bushels of corn, which was $33\frac{1}{3}\%$ of his entire crop. How many bushels did he raise?

12. A real-estate broker sold a lot for \$8400, which was $116\frac{2}{3}\%$ of the cost of it. How much did it cost?

13. A horse was sold for \$320, which was but 80% of what he cost. How much did he cost?

14. A house was sold for \$8100, which was $112\frac{1}{2}\%$ of the cost. What was the cost?

15. A man increased his capital $8\frac{1}{3}\%$ and then had \$16900. How much had he at first?



INTEREST

293. 1. When a sum equal to 5% of the amount of money lent is paid for the use of it for 1 year, how much will be paid for the use of \$100 for 1 year? For 2 years?

2. When the allowance for the use of money is 6% per year, what is the allowance for the use of \$100 for 1 year? For 2 years? For 3 years? For $3\frac{1}{2}$ years?

3. When the sum paid for the use of money is 8% per year, what must be paid per year for \$50? For \$500?

4. When the sum paid for the use of money is 12% yearly, what must be paid for the use of \$100 for 1 year?

5. When the allowance for the use of money is 8% per year, what must be paid for the use of \$100 for 6 months? For 1 month? For $\frac{1}{2}$ month? For $\frac{1}{4}$ month? For $\frac{1}{8}$ month? For 10 days? For 20 days?

6. When 6% is paid per year for the use of money, how much will \$500 amount to in 2 years? In 3 years?

7. When \$500 is loaned for $1\frac{1}{2}$ years at 8% per year, what will be the amount?

294. Interest is the sum paid for the use of money.

295. The Principal is the sum for the use of which interest is paid.

296. The Rate of interest is the annual rate per cent.

297. The Amount is the sum of principal and interest.

ORAL EXERCISES.

298. 1. What is the interest of \$200 for 1 year at 5%?
 2. What is the interest of \$300 for 1 year at 8%?
 3. What is the interest of \$200 for 2 years at 5%?
 4. What is the interest of \$500 for 2 years at 6%?
 5. What is the interest of \$400 for $1\frac{1}{2}$ years at 4%?
 6. What is the interest of \$600 for $1\frac{1}{2}$ years at 6%?
 7. What is the interest of \$400 for $\frac{1}{2}$ year at 6%?
 8. What is the interest of \$400 for 3 months at 6%?
 9. What is the interest of \$200 for 1 month at 6%?
 10. What is the amount of \$100 for 2 years at 6%?

WRITTEN EXERCISES.

299. 1. Find the interest of \$234.27 for 2 yr., 7 mo., 12 da. at 6%.

PROCESS.

$$\begin{array}{r}
 \$234.27 \\
 .06 \\
 \hline
 12) \$14.0562 \text{ Int. for 1 yr.} \\
 \underline{\$1.1713} \text{ Int. for 1 mo.} \\
 31.4 \\
 \hline
 \$36.78+ \text{ Int. 2 yr., 7 mo., 12 da.}
 \end{array}$$

ANALYSIS.—Since the rate of interest is 6%, we find 6% of \$234.27, which is \$14.0562, the interest for one year.

Since there are 12 mo. in a year, the interest for 1 month is found by dividing \$14.0562 by 12,

which gives \$1.1713, the interest for 1 month.

Since in 2 yr., 7 mo. there are 31 months, and 12 days are $\frac{1}{3}$, or .4, of a month, the entire interest is 31.4 times \$1.1713, the interest for 1 month. 31.4 times \$1.1713 is \$36.78+, the entire interest.

RULE.—*Find the interest for 1 year at the given rate, and then for 1 month by dividing by 12. Multiply this quotient by the years and months reduced to months, and the days to the fraction of a month.*

2. Find the interest of \$32.16 for 1 yr., 6 mo. at 6%.
3. Find the interest of \$28.32 for 2 yr., 3 mo. at 6%.
4. Find the interest of \$19.24 for 1 yr., 8 mo. at 6%.
5. Find the interest of \$16.36 for 1 yr., 5 mo. at 7%.
6. Find the interest of \$48.50 for 2 yr., 3 mo. at 8%.
7. Find the interest of \$20.00 for 3 yr., 2 mo. at 4%.
8. Find the interest of \$31.69 for 2 yr., 5 mo. at 7%.
9. Find the interest of \$42.65 for 1 yr., 8 mo. at $8\frac{1}{2}\%$.
10. Find the interest of \$31.26 for 1 yr., 4 mo. at 9%.
11. Find the interest of \$57.36 for 2 yr., 8 mo. at 7%.
12. Find the interest of \$38.24 for 6 mo., 12 da. at 8%.
13. Find the interest of \$27.16 for 5 mo., 10 da. at 8%.
14. Find the interest of \$32.42 for 7 mo., 15 da. at 9%.
15. Find the interest of \$28.16 for 8 mo., 9 da. at 6%.
16. Find the interest of \$64.24 for 3 mo., 10 da. at 5%.
17. Find the interest of \$58.22 for 2 mo., 8 da. at 6%.
18. Find the interest of \$56.89 for 2 yr., 3 mo., 6 da. at 8%.
19. Find the interest of \$38.65 for 3 yr., 2 mo., 12 da. at 9%.
20. Find the interest of \$42.72 for 4 yr., 1 mo., 10 da. at 7%.
21. Find the interest of \$83.26 for 3 yr., 5 mo., 7 da. at 6%.
22. Find the amount of \$24.71 for 1 yr., 5 mo., 6 da. at 8%.
23. Find the amount of \$39.42 for 27 da. at 10%.
24. Find the amount of \$68.73 for 21 da. at 8%.
25. Find the amount of \$69.86 for 13 da. at 7%.
26. Find the amount of \$38.41 for 17 da. @ 7%.
27. If I lend \$1000, how much will be due me in 2 yr., 5 mo., 18 da., interest at 6%?
28. What is the interest of \$645 from Sept. 15, 1880, to July 5, 1882, interest at 7%?

QUESTIONS FOR REVIEW.

What is per cent.? What is percentage? What is the sign of per cent.? In what ways may per cent. be expressed? What is the base? What is the rate? What is the percentage? When the base and rate are given, how is the percentage found? When the percentage and base are given, how is the rate found? When the percentage and rate are given, how is the base found?

What is interest? What is the principal? What is the rate? What is the amount? How is interest computed?

GENERAL REVIEW.

300. 1. What will be the cost of $8\frac{3}{4}$ yd. of cloth at $\$5\frac{3}{4}$ per yd.?

2. A drover bought 24 head of cattle at \$31 per head and 40 at \$25.75 per head. How much did they cost him?

3. The Declaration of Independence was signed July 4, 1776. How long ago was that?

4. How many bushels of oats at \$.30 per bushel can be bought for \$36.24?

5. If I paid \$93.50 for 34 yards of cloth, what was the price per yard?

6. What will be the cost of 9 pieces of prints, each containing $34\frac{1}{4}$ yards, at 7 cents per yard?

7. How many square yards of carpeting will be required to cover a floor 18 feet wide and 24 feet long?

8. A man had a rectangular field which was 60 rods long and 35 rods wide. How many acres did it contain?

9. A lad spent 45 cents, which was $\frac{3}{4}$ of all the money he had. How much money had he?

10. James was only $\frac{3}{4}$ as old as his mother, yet James was 21 years old. How old was his mother?

11. A ladder which was 42 feet long was found to be just

$\frac{3}{4}$ of the height of a liberty pole. How high was the liberty pole?

12. Two boys have together 44 cents, but one of them has 4 cents more than the other. How much has each?

13. There were two baskets which together contained 27 apples, but one of them contained 7 more than the other. How many apples did each contain?

14. A gentleman gave some boys 5 cents each. If he had given them 7 cents each it would have taken 12 cents more. How many boys were there?

15. A can mow a field in 5 days and B can mow it in 4 days. If they work together, in how many days can they mow it?

16. James can do a piece of work in $5\frac{1}{2}$ days and Henry can do it in $6\frac{1}{2}$ days. If they work together, in how many days can they do it?

17. A merchant sold some cloth for \$75 and thereby lost $\frac{3}{8}$ of what it cost. How much did it cost?

18. By selling a quantity of stoves for \$490 I gained $\frac{1}{4}$ of what they cost me. How much did they cost me?

19. What is the interest of \$364.18 for 2 yr., 3 mo., 15 da. at 7%?

20. I loaned \$325 at 7%. How much was due me in 2 yr., 4 mo., 5 da.?

21. Mr. B borrowed \$316.25 at 6% Jan. 4, 1879, and paid the sum with interest Apr. 7, 1881. How much did he pay?

22. A grocer sold 32 gal., 3 qt., 1 pt. of molasses in January; 42 gal., 1 qt., 1 pt. in February, and 26 gal., 3 qt., 1 pt. in March. How much did he sell in the three months?

23. If he had 120 gallons on hand on January 1, how much had he left April 1?

24. A stationer sold 4 reams, 8 quires, 17 sheets of paper in one day. How many sheets did he sell?

25. A farmer who had 192 sheep sold $6\frac{1}{4}\%$ of them to a butcher. How many did he sell?

26. A merchant who had 483 yards of cloth sold $14\frac{3}{4}\%$ of it at \$2.50 per yard. How much did he receive for it?

27. After spending 35% of his money in traveling, a young man found that he had \$130 left. How much had he at first?

28. A man who had 376 sheep sold 25% of the flock at \$4.50 per head. How much did he receive for them?

29. If $\frac{3}{4}$ of a yard of broadcloth costs \$3.90, what will 12 yards cost?

30. If $\frac{5}{8}$ of an acre of land produces $18\frac{3}{4}$ bushels of wheat, how many bushels will $9\frac{1}{2}$ acres produce?

31. If a man smokes 5 cigars per day for 15 years, how much will he pay for cigars in that time, assuming that they cost him 7 cents each, and calling 365 days a year?

32. A man who owned $\frac{3}{4}$ of an acre of land sold $\frac{1}{4}$ of it for \$999. How much would an acre be worth at that rate?

33. The smaller of two fractions is $\frac{1}{2}\frac{1}{3}$ and the difference between them is $\frac{7}{6}$. What is the other?

34. A sum of money was divided among five persons as follows: A received $\frac{1}{5}$, B $\frac{1}{4}$, C $\frac{1}{10}$, D $\frac{1}{20}$ of it, and E the remainder, which was \$2000. What was the sum divided?

35. What number divided by $1\frac{2}{3}$ will give a quotient of $3\frac{2}{3}$?

36. The product of two numbers is 8, and $\frac{2}{3}$ of one of them is $\frac{8}{9}$. What is the other?

37. After spending \$5 more than $\frac{2}{3}$ of his money, a man found that he had \$15 left. How much money had he at first?

38. A boy lost $\frac{1}{2}$ of his money and then earned 30 cents, when he had just $\frac{1}{2}$ as much as he had at first. How much had he at first?

39. Mr. Henry C. Dayton bought of James Davis & Co., Apr. 19, 1882, 16 yd. of broadcloth @ \$5.20 per yard, 2 yd. muslin @ 16 cents per yard, 14 yd. linen @ \$.80 per yard,

and 36 yd. cassimere at \$1.75 per yard. Make out the bill in proper form and receipt it.

40. If it takes a man 4 hr., 30 min. to mow an acre of grass, how long will it take him to mow a field of 20 acres, if he works 10 hours per day?

41. A man who had \$8000, invested $37\frac{1}{2}\%$ in a house, and loaned the rest on interest at 6%. How much interest was due him in 5 mo., 8 da.?

42. If 35 bushels of oats will keep 30 horses 1 week, how many weeks will 350 bushels keep 7 horses?

43. A merchant put $\frac{2}{3}$ of his money in a bank, invested $\frac{1}{5}$ of it in bonds, $\frac{1}{10}$ of it in a manufactory, and had \$7000 left. How much had he at first?

44. The sum of two numbers is 140, one of which is $\frac{3}{4}$ of the other. What are the numbers?

45. A, B, and C can together do a piece of work in 10 days. If A can do it alone in 30 days, and B can do it in 25 days, in what time can C do it alone?

46. A man spent $\frac{3}{4}$ of $37\frac{1}{2}\%$ of his annual salary, and had \$2300 left. How much was his annual salary?

47. A man who owned 60% of a vessel sold $33\frac{1}{3}\%$ of his share for \$12500. What would the vessel be worth at that rate?

48. By selling goods at \$1.86 per yard, a merchant gained 20% of the cost of them. How much did they cost per yard?

49. A horse was sold for \$215, the seller losing $37\frac{1}{2}\%$ of what he paid for him. How much did he pay for him?

50. A nursery man sold 22250 apple trees. This number was 45% of the number of trees in 45% of the number of rows of trees. How many apple trees had he?

ANSWERS

Page 30.

2. 1035.
3. 802.
4. 1298.
5. 1437.
6. 1329.
7. \$1396.
8. \$76.47.
9. \$450.89.
10. \$274.91.

Page 31.

11. 14898 horses.
12. 8314 gallons.
13. 4384 hundredths.
14. 340 rods.
15. 22220.
16. 11148.
17. \$121.87.
18. \$40.52.
19. \$103.535.
20. \$981.815.
21. 17103.
22. 117940.
23. 32282435.
24. \$161.76.
25. \$196.16.
26. \$59.133.
27. \$800.

Page 32.

23. 1065 acres.
29. 2437 in first.
1661 in second.
4098 in both.
30. 10636.
31. 2391 miles.
32. 3255 bushels.

33. 1164 miles.
34. 110 acres.
35. \$16409.
36. \$11000.

Page 33.

37. 3381 potatoes.
38. \$10500.
39. 810 acres
1620 acres.
40. 20985 books.
41. 2879 miles.
42. 102970 lbs.
43. \$17124.
44. 905 fish.
45. 5924 grains.
46. \$13400.
47. 254554 feet.

Page 34.

48. \$14755.
49. 1361 teachers.
50. 4439 miles.
51. 11794.
52. 11373.
53. 116982.
54. 107759.

Page 45.

2. 169.
3. 202.
4. 291.
5. 329.
6. 264.
7. 287.
8. 115.
9. \$20.39.

10. \$7.92.
11. \$16.48.
12. \$21.05.
13. \$9.25.
14. \$35.38.
15. \$45.91.
17. 953.
18. 2041.
19. 1546.
20. 1317.
21. 36544.

Page 46.

22. 291.
23. 507.
24. 387.
25. 152.
26. 1991.
27. 4886.
28. 1503.
29. 1852.
30. 2328.
31. 44139.
32. 24472.
33. 440100.
34. 350890.
35. 4180175.
36. 42581.
37. \$2229.
38. \$7960.08.
39. 12797 feet.
40. \$4825.75.
41. 13006.
42. \$18550.
43. 18309 cattle.
44. 34711 stamps.
45. \$4294.04.
46. \$2518.

Page 49.

1. 8432.
2. 1185.
3. 2377.
4. 28583.
5. 11388.
6. 2107.
7. \$17685.
8. 1882.
9. \$12148.
10. \$7625.
11. \$619.
12. 98 yards.
13. 43 sheep.
14. \$60.
15. \$772.

Page 50.

16. \$1007.
17. 915 A.
1605 B.
18. 13689 men.
19. 1486 bushels.
20. 5150 sq. mi.
21. \$9999.10.
22. 66883.

Page 53.

2. 944.
3. 1230.
4. 1287.
5. 2478.
6. 948.
7. 1788.
8. 2345.
9. 3328.
10. 888.
11. 28956.
12. 39170.
13. 35562.
14. 48888.
15. 41880.
16. 30072.
17. 19251.
18. 18768.
19. 34024.
20. 148065.
21. 281637.
22. 255702.

23. 295587.
24. 173576.
25. 220486.
26. 210834.
27. 341400.
28. 205632.
29. \$38.25.
30. \$128.10.
31. \$138.75.
32. 16135 emigrants.
33. \$193.41.
34. \$58120.
35. 49912 men.
36. 238275 papers.
37. 42240 feet.
38. \$110.
39. 11016 bricks.
40. \$254.10.

Page 57.

41. \$109.74.
42. \$54.80.
43. \$41562.

Page 59.

11. 7680.
12. 14070.
13. 13700.
14. 39760.
15. 11840.
16. 119200.
17. 149100.
18. 397600.
19. 92200.
20. 493800.

Page 61.

4. 8608.
5. 11640.
6. 23562.
7. 31175.
8. 42952.
9. 32445.
10. 19824.
11. 50270.
12. 27027.
13. 30144.
14. 41097.
15. 458481.
16. 645650.

17. 802497.
18. 126028.
19. 192786.
20. 135945.
21. 96624.
22. \$821.80.
23. \$1091.43.
24. \$943.02.
25. \$1047.05.
26. \$1500.85.
27. \$3285.08.
28. 3977790.
29. 11884020.
30. 14011375.
31. 12316256.
32. 6742220.
33. 11387368.
34. 11616215.
35. 10213954.
36. 999977164.
37. 2067421020.
38. 4133249372.
39. 3515881096.
40. 221461306500.
41. 7347 miles.

Page 62.

42. \$41160.
43. 136032 tickets.
44. \$1053486.
45. \$5380.99.
46. 195000 pounds.
47. 118080 sheets.
48. \$8400.
49. 2688 miles.
50. 8532 bushels.

Page 64.

1. \$61.22.
2. 390 trees.
3. \$1560.
4. \$720.
5. \$4430.40.
6. \$2567.76.
7. \$2353.50.
8. \$313.55.

Page 65.

9. \$1209.

10. \$134.94.
11. 315 miles.
12. \$18.33.
13. \$90.75.
14. \$2380, loss.
15. \$2630.
16. \$755.25, gain.
17. \$5219.

Page 74.

2. 284.
3. 427.
4. 427.
5. 229.
6. 531.
7. 397.
8. 264.
9. 469.
10. 279.
11. 335.
12. 357.
13. 471.

Page 75.

15. 1218.
16. 1366.
17. 496.
18. 597.
19. 1545.
20. 984.
21. 406.
22. 876.
23. 554.
24. 1353.
25. 548.
26. 1234.
27. 913.
28. 1488.
29. 908.
30. 2296.
31. 12052.
32. 4857.
33. \$4.87.
34. \$4.54.
35. \$2.37.
36. \$3.91.
37. \$4.59.
38. \$3.03.
39. 166 plows.

40. 225 coats.
41. 156 hours.
42. 211 weeks.
43. \$753.
44. 392 hours.
45. 287 days.
46. 689 persons.
47. \$1765.
48. \$5739.
49. 3271.

Page 78.

13. 227.
14. 74.
15. 12.
16. 12.
17. 44.
18. 28.
19. 10.
20. 7.
21. 10.
22. 10.
23. 11.
24. 11.

Page 79.

26. 679.
27. 382.
28. 213.
29. 432.
30. 306.
31. 641.
32. 65.
33. 55.
34. 48.
35. 67.
36. 55.
37. 48.
38. 73.
39. 57.

Page 80.

40. 82.
41. 42.
42. 37.
43. 36.
44. 29.
45. 723.
46. 606.
47. 918.

48. 520.
49. 541.
50. 723.
51. 616.
52. 752.
53. 68.
54. 42.
55. 97.
56. 665.
57. 777.
58. 582.
59. 672.
60. 561.
61. 306.
62. 682.
63. 417.
64. 862.
65. 453.
66. 346.
67. 519.
68. 2134.
69. 725.
70. 561.
71. 601.
72. 309.
73. 468.
74. 312.
75. 580.
76. \$61.
77. 34 barrels.
78. 60 bushels.
79. \$3080.
80. \$51613.
81. 168 gallons.

Page 81.

82. 97 schooners.
83. 45 cars.
84. 230 days.
85. 5130 shares.
86. 5.

Page 83.

1. 117.
2. 40.
3. 112.
4. 56.
5. 109.
6. 60.

Page 84.

10. 61600 yards.
11. 7252 pounds.
12. \$4000.
13. \$396.
14. \$1410.
15. \$4725.
16. 4980 pounds.
17. \$510.
18. \$198.
19. 442 B's.
1284 C's.
1511 D's.
20. 804 miles.
21. 76 barrels.

Page 85.

22. \$5.
23. 8 years.
24. \$400, loss.

Page 88.

2. $2^3, 3, 7$.
3. $2^5, 7$.
4. $2^4, 3^2$.
5. $2, 3^2, 11$.
6. $2^3, 3^3$.
7. $2^2, 11^2$.
8. $2^6, 3^2$.
9. $2^4, 3^2$.
10. $2^5, 3, 7$.
11. $2^2, 3^2, 11$.
12. 5^4 .
13. $2^4, 3, 19$.
14. $2^6, 13$.
15. $2, 3^2, 5, 11$.
16. $2^4, 3^2, 7$.
17. $2^5, 5, 7$.
18. $2^3, 3, 7^2$.
19. $2, 5^4$.
20. 2^{10} .
21. $2^6, 3^3$.
22. $2^8, 5$.
23. $2^7, 3^2$.
24. $2^2, 3^2, 89$.
25. $2^2, 5^4$.
26. $2^4, 3^4$.
27. $2^4, 3, 5, 7$.
28. $2^5, 3^2, 7$.

29. $2^3, 3^2$.
30. $2^2, 3, 203$.
31. $2, 5, 289$.

Page 90.

2. $2\frac{1}{2}$.
3. $1\frac{1}{2}$.
4. 4.
5. 3.
6. 16.
7. $27\frac{1}{2}$.
8. 40.
9. $18\frac{1}{2}$.
10. 45.
11. 6.
12. $7\frac{1}{2}$.
13. 4.
14. 60.
15. 40.
16. $3\frac{1}{2}$.
17. $16\frac{1}{2}$.
18. $6\frac{1}{2}$.
19. 8.
20. $2\frac{1}{2}$.
21. 1.
22. 3.
23. $6\frac{1}{2}$.
24. $2\frac{1}{2}$.
25. \$3.12.

Page 91.

26. \$48.
27. \$0.14.
28. \$48.
29. $53\frac{1}{2}$ yards.

Page 96.

2. $\frac{1}{2}$.
3. $\frac{1}{3}$.
4. $\frac{1}{4}$.
5. $\frac{1}{5}$.
6. $\frac{1}{6}$.
7. $\frac{1}{7}$.
8. $\frac{1}{8}$.
9. $\frac{1}{9}$.
10. $\frac{1}{10}$.
11. $\frac{1}{11}$.
12. $\frac{1}{12}$.
13. $\frac{1}{13}$.

14. $\frac{1}{14}$.
15. $\frac{1}{15}$.
16. $\frac{1}{16}$.
17. $\frac{1}{17}$.
18. $\frac{1}{18}$.
19. $\frac{1}{19}$.
20. $\frac{1}{20}$.
21. $\frac{1}{21}$.
22. $\frac{1}{22}$.
23. $\frac{1}{23}$.
24. $\frac{1}{24}$.
25. $\frac{1}{25}$.
26. $\frac{1}{26}$.
27. $\frac{1}{27}$.
28. $\frac{1}{28}$.
29. $\frac{1}{29}$.

Page 97.

2. $7\frac{1}{2}$.
3. $1\frac{1}{2}$.
4. $10\frac{1}{2}$.
5. $7\frac{1}{2}$.
6. $10\frac{1}{2}$.
7. $7\frac{1}{2}$.
8. $10\frac{1}{2}$.
9. $2\frac{1}{2}$.
10. $2\frac{1}{2}$.
11. $3\frac{1}{2}$.
12. $7\frac{1}{2}$.
13. $1\frac{1}{2}$.
14. $11\frac{1}{2}$.
15. $7\frac{1}{2}$.
16. $16\frac{1}{2}$.
17. $21\frac{1}{2}$.
18. $21\frac{1}{2}$.
19. $18\frac{1}{2}$.
20. $5\frac{1}{2}$.
21. $10\frac{1}{2}$.

Page 98.

2. 45.
3. 54.
4. $49\frac{1}{2}$.
5. $38\frac{1}{2}$.
6. $61\frac{1}{2}$.
7. $16\frac{1}{2}$.
8. $16\frac{1}{2}$.
9. $13\frac{1}{2}$.
10. $25\frac{1}{2}$.

11. 192 $\frac{1}{2}$.
12. 162 $\frac{1}{2}$.
13. 32 $\frac{1}{2}$.
14. 192 $\frac{1}{2}$.
15. 94 $\frac{1}{2}$.
16. 202 $\frac{1}{2}$.
17. 167.
18. 192 $\frac{1}{2}$.
19. 33 $\frac{1}{2}$.
20. 34 $\frac{1}{2}$.
21. 72 $\frac{1}{2}$.

Page 100.

2. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
3. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
4. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
5. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
6. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
7. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
8. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
9. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
10. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
11. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
12. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.
13. 1 $\frac{1}{2}$, 3 $\frac{1}{2}$, 5 $\frac{1}{2}$.

Page 101.

15. 3 $\frac{1}{2}$, 5 $\frac{1}{2}$, 7 $\frac{1}{2}$.
16. 3 $\frac{1}{2}$, 5 $\frac{1}{2}$, 7 $\frac{1}{2}$.
17. 3 $\frac{1}{2}$, 5 $\frac{1}{2}$, 7 $\frac{1}{2}$.
18. 3 $\frac{1}{2}$, 5 $\frac{1}{2}$, 7 $\frac{1}{2}$.
19. 7 $\frac{1}{2}$, 9 $\frac{1}{2}$, 11 $\frac{1}{2}$.
20. 20 $\frac{1}{2}$, 22 $\frac{1}{2}$, 24 $\frac{1}{2}$.
21. 10 $\frac{1}{2}$, 12 $\frac{1}{2}$, 14 $\frac{1}{2}$.
22. 20 $\frac{1}{2}$, 22 $\frac{1}{2}$, 24 $\frac{1}{2}$.
23. 10 $\frac{1}{2}$, 12 $\frac{1}{2}$, 14 $\frac{1}{2}$.

Page 104.

2. 1 $\frac{1}{2}$.
3. 1 $\frac{1}{2}$.
4. 1 $\frac{1}{2}$.
5. 2 $\frac{1}{2}$.
6. 1 $\frac{1}{2}$.
7. 1 $\frac{1}{2}$.
8. 1 $\frac{1}{2}$.
9. 1 $\frac{1}{2}$.
10. 14 $\frac{1}{2}$.
11. 18 $\frac{1}{2}$.
12. 20 $\frac{1}{2}$.

13. 22 $\frac{1}{2}$.
14. 24 $\frac{1}{2}$.
15. 22 $\frac{1}{2}$.
16. 17 $\frac{1}{2}$.
17. 23 $\frac{1}{2}$.
18. \$85 $\frac{1}{2}$.
19. \$351 $\frac{1}{2}$.
20. \$78 $\frac{1}{2}$.
21. 75 $\frac{1}{2}$ miles.
22. 115 $\frac{1}{2}$ acres.

Page 107.

3. 1.
4. 1.
5. 1.
6. 1.
7. 1.
8. 1.
9. 1.
10. 1.
11. 1.
12. 1.
13. 1.
14. 1.
15. 1.
16. 5.
17. 5.
18. 7.
19. 8.
20. 6.
21. 13.
22. 5.
23. 1.
24. 1.
25. 1.
26. 1.
27. 1.
28. 5.
29. 8.
30. 12.
31. 21.
32. 2.
33. 66 $\frac{1}{2}$ acres.
34. \$41 $\frac{1}{2}$.
35. \$193 $\frac{1}{2}$.
36. \$902 $\frac{1}{2}$.
37. \$97 $\frac{1}{2}$.
38. \$391 $\frac{1}{2}$.

Page 108.

39. \$28 $\frac{1}{2}$.
40. \$12 $\frac{1}{2}$.

Page 110.

2. 1 $\frac{1}{2}$.
3. 2.
4. 3.
5. 3.
6. 3.
7. 3.
8. 5.
9. 3.
10. 22.
11. 19 $\frac{1}{2}$.
12. 110 $\frac{1}{2}$.
13. 200 $\frac{1}{2}$.
14. \$16 $\frac{1}{2}$.
15. \$41 $\frac{1}{2}$.
16. \$28 $\frac{1}{2}$.
17. 206 miles.
18. \$84.
19. 121 $\frac{1}{2}$ hours.
20. \$88 $\frac{1}{2}$.
21. \$88 $\frac{1}{2}$.

Page 111.

22. \$825.
23. \$125 $\frac{1}{2}$.
24. \$60 $\frac{1}{2}$.

Page 112.

2. 10.
3. 21.
4. 7 $\frac{1}{2}$.
5. 3 $\frac{1}{2}$.
6. 18.
7. 228.
8. 26.
9. 17 $\frac{1}{2}$.
10. 64.
11. 38 $\frac{1}{2}$.
12. 19 $\frac{1}{2}$.
13. 19 $\frac{1}{2}$.
14. 69.
15. 110.
16. 123.
17. 173 $\frac{1}{2}$.
18. 253 $\frac{1}{2}$.
19. 575 $\frac{1}{2}$.

Page 113.

20. \$4380.
 21. \$156.
 22. \$6758½.
 23. 286½ miles.
 24. \$225.
 25. \$85½.
 26. \$139½.

Page 115.

2. ½.
 3. ½.
 4. ½.
 5. ¾.
 6. ¾.
 7. ¾.
 8. ¾.
 9. ¾.
 10. ¾.
 11. ¾.
 12. ¾.
 13. ¾.
 14. ¾.
 15. ¾.
 16. ¾.
 17. ¾.
 18. ¾.
 19. ¾.
 20. 1½.
 21. 4½.
 22. 3½.
 23. ¾.
 24. 232½ acres.
 25. \$11½.
 26. 57½ feet.
 27. 168½ acres.
 28. \$145½.

Page 116.

29. \$177½.
 30. \$156½.
 31. 1794½ bushels.

Page 118.

2. ¾.
 3. ¾.
 4. ¾.
 5. ¾.
 6. ¾.

7. ¾.
 8. ¾.
 9. ¾.
 10. ¾.
 11. ¾.
 12. ¾.
 13. ¾.
 15. ¾.
 16. ¾.
 17. ¾.
 18. ¾.
 19. ¾.
 20. ¾.
 21. ¾.
 22. ¾.
 23. ¾.
 24. \$1½.
 25. \$11½.
 26. \$1½.
 27. \$1½.
 28. \$155½.
 29. \$91½.

Page 120.

2. 20.
 3. 81.
 4. 49.
 5. 40.
 6. 72.
 7. 21½.
 8. 26½.
 9. 65.
 10. 64½.
 11. 180.
 12. 71½.
 13. 33½.
 14. 37½.
 15. 60½.
 16. 22½.
 17. 8½.
 18. 6½.
 19. 7½.
 20. 4½.
 21. 12.
 22. 16 lots.
 23. 24 peaches.
 24. 4½ tons.
 25. 7½ cords.
 26. 10 pairs.

27. 12 pictures.
 28. 15 barrels.

Page 122.

2. ¾.
 3. ¾.
 4. ¾.
 5. ¾.
 6. ¾.
 7. ¾.
 8. ¾.
 9. ¾.
 10. ¾.
 11. ¾.
 12. 1.
 13. ¾.
 14. ¾.
 15. ¾.
 16. ¾.

Page 123.

18. ¾.
 19. 2½.
 20. 2½.
 21. 2.
 22. 1½.
 23. 1.
 24. ¾.
 25. 18½ days.
 26. 7½ pounds.
 27. 5½ acres.
 28. 8 barrels.
 29. 6 plows.
 30. 10 books.
 31. 7 diaries.
 32. 7½ dozen.

Page 124.

34. ¾.
 35. ¾.
 36. ¾.
 37. ¾.
 38. ¾.
 39. 11½.
 40. ¾.
 41. ¾.
 42. 1½.
 43. ¾.
 44. ¾.
 45. 2½.

Page 125.

2. 140.
3. 108.
4. 135.
5. 126.
6. 143.
7. 150.
8. 120.
9. 120.
10. 143.
11. 204.
12. $2\frac{1}{2}$.
13. $1\frac{1}{2}$.
14. $1\frac{1}{17}$.
15. $1\frac{1}{15}$.
16. $1\frac{1}{2}$.
17. \$64.
18. \$150.
19. \$91.
20. 40 mi.
21. 280 acres.
22. \$450.

Page 128.

1. $34\frac{1}{11}$.
2. $\$18\frac{1}{10}$.
3. $1\frac{1}{10}$.
4. $\frac{1}{10}$.
5. $4\frac{1}{10}$.
6. $4\frac{7}{10}$.
7. $2\frac{1}{10}$.
8. $13\frac{1}{2}$ bar.
9. \$84.
10. $140\frac{1}{2}$ acres.
11. \$6434 $\frac{1}{2}$.
12. \$266 $\frac{1}{2}$.
13. $\frac{1}{15}$.

Page 129.

14. 60.
15. \$14727 $\frac{1}{11}$.
16. \$19285 $\frac{1}{2}$.
17. 48 yards.
18. $2\frac{1}{11}$ days.
19. $1\frac{1}{15}$ days.
20. $4\frac{1}{10}$ days.
21. $11\frac{1}{2}$ days.
22. 24 days.

23. \$160, James.
\$240, brother.
24. \$372 $\frac{1}{2}$, one.
25. \$497 $\frac{1}{2}$, other.
125 boys.
175 girls.

Page 130.

26. \$80.
27. 30 days, A.
15 days, B.
28. \$50000.
29. 1.
30. 40 marbles.

Page 136.

2. $\frac{1}{10}$.
3. $\frac{1}{10}$.
4. $\frac{1}{10}$.
5. $\frac{1}{10}$.
6. $\frac{1}{10}$.
7. $\frac{1}{10}$.
8. $\frac{1}{10}$.
9. $\frac{1}{10}$.
10. $\frac{1}{10}$.
11. $\frac{1}{10}$.
12. $\frac{1}{10}$.
13. $\frac{1}{10}$.
14. $\frac{1}{10}$.
15. $\frac{1}{10}$.
16. $\frac{1}{10}$.
17. $\frac{1}{10}$.

Page 137.

2. .375.
3. .875.
4. .3125.
5. .95.
6. .04.
7. .32.
8. .025.
9. .0125.
10. .52.
11. .85.
12. .35.
13. .555 +.
14. .024.
15. .01066 +.

16. .42857 +.
17. .9375.

2. 42.086.
3. 34.0735.
4. 38.421.
5. 51.583.
6. 23.6818.
7. 52.36.
8. 53.8264.
9. 38.2395.
10. 69.353.
11. 390.0795.
12. 303.032.
13. 309.02.
14. 240.4584.
15. 70.645.

Page 138.

16. 1.081.
17. 1.469853.
18. \$1607.28.
19. 167.92 mi.

Page 139.

2. 11.93.
3. 14.915.
4. 17.945.
5. 19.47.
6. 23.175.
7. 20.7444.
8. 19.657.
9. 21.428.
10. 3.0004.
11. 4.3464.
12. 3.2364.
13. .64935.
14. .017874.
15. .0514.
16. 4.2804.
17. 2.8928.
18. 1.39455.
19. 2.6466.
20. 2.45984.
21. \$7025.66.
22. \$4570.11.

Page 140.

2. .1184.
3. 1.008.
4. 15.68.
5. .2196.
6. .03772.
7. .287624.
8. 2.16975.
9. 1.5228.
10. .7378.

Page 141.

11. .018088.
12. .514098.
13. 91.125.
14. .4225.
15. .0027608.
16. .0147396.
17. .00090006.
18. .000617945.
19. .0443196.
20. .088136.
21. .0613632.
22. 72.81792.
24. 286.3.
25. 3829.5.
26. 2835.
27. 38.4.
28. 52.76.
29. 8432.5.
30. \$854.8875.
31. \$80.0125.
32. \$38.9025.

Page 143.

2. .36.
3. 2.5.
4. 36.4.
5. 3.27.
6. .516.
7. 64.2.
8. 5.27.
9. .265.
10. .342.
11. .066637 +.
12. 2.34.
13. 6.82.
14. 426.
15. 3.282.

16. 32.4.
17. 143.69 +.
18. 5.2516 +.
19. 4.835 +.
20. 826.625 +.
21. 67563.52 +.
22. 122.425 +.
23. .060204 +.
24. .87444 +.
25. 10000000000.
27. 3.925.
28. 2.645.
29. .3695.
30. .04825.
31. .03862.
32. .004285.
33. \$2.25.

Page 144.

34. 28 bushels.
35. 23.683 + mi.
36. \$32.55.
37. \$3.18.

Page 149.

1. \$524.22.
2. \$113.95.
3. \$50.277.
4. \$476.98.
5. \$651.91.
6. \$848.94.
7. \$325.25.
8. \$223.70.
9. \$.166 +.
10. \$14.875.
11. \$100.20.
12. 52 pairs.

Page 150.

13. \$19.125.
14. \$4.45.
15. \$14.06.
16. \$.31½.
17. \$245.75.
18. 40½ mo.
19. \$3.40.
20. \$918.816 +.
21. \$.065.
22. \$1.60.
23. \$347.75.

24. \$338.
25. \$803.75.

Page 153.

2. \$270.56.
3. \$2717.60.
4. \$2820.97.

Page 154.

5. \$28.45.
6. \$1520.92.
7. \$334.54.
8. \$245.80.

Page 166.

2. 140 quarts.
3. 208 inches.
4. 55 pints.
5. 8435 seconds.
6. 37478 ounces.
7. 294 hours.
8. 31872 grains.
9. 235 pints.
10. 536 inches.
11. 12111 sq. in.
12. 10592½ sq. yd.
13. 107328 cu. in.
14. 69094 ounces.
15. 27010 grains.
16. 7900 grains.
17. 1692 sheets.
18. 27 inches.
19. 7200 minutes.
20. 156 gills.
21. 631 hours.
22. 967 inches.
23. 157 quarts.
24. 21½ quarts.
25. 14 hr., 24 min.

Page 167.

2. 21 bu., 1 pk., 4 qt.
3. 8 rd., 1 ft., 9 in.
4. 295 gal., 2 qt.
5. 4 hr., 42 min., 46 sec.
6. 1 T., 42 lb., 12 oz.
7. 10 wk., 2 da., 1 hr.
8. 1 lb., 4 oz., 15 gr.

9. 1 lb., 6 dr., 2 sc., 5 gr.
10. 2 sq. yd., 7 sq. ft., 84 sq. in.

Page 168.

11. 145 rd., 2 yd., 1 ft., 2 in.
12. 7 sq. rd., 6 sq. ft., 77 sq. in.
13. 6 reams, 17 quires, 9 sheets.
14. 2006 gal., 2 qt., 1 pt., 1 gi.
15. 2 mi., 106 rd., 5 yd., 2 ft.

2. 23 gal., 3 qt.
3. 30 lb., 9 oz., 15 pwt.
4. 49 bu., 3 pk., 7 qt.
5. 44 da., 20 hr., 28 min., 55 sec.
6. 128 A., 40 sq. rd.

Page 169.

7. 87 cu. yd., 25 cu. ft.
8. 22 T., 14 cwt., 65 lb., 2 oz.
9. 14 lb., 8 oz., 7 dr., 14 gr.

2. 4 gal., 3 qt., 1 pt., 3 gi.
3. 6 lb., 7 oz., 1 pwt., 22 gr.
4. 8 bu., 3 pk., 5 qt.

Page 170.

5. 5 da., 3 hr., 59 min., 11 sec.
6. 102 A., 141 sq. rd.
7. 7 C., 118 cu. ft.
8. 70 bu., 1 pt.
9. 3 T., 1 cwt., 14 oz.
10. 4 reams, 17 quires, 16 sheets.
11. 4 sq. yd., 136 sq. in.

12. 4 rd., 1 yd., 2 ft., 10 in.
13. 16 lb., 6 oz., 1 dr.
14. 4 T., 17 cwt., 5 lb., 10 oz.
16. 36 yr., 2 mo., 28 days.
17. 28 yr., 2 mo., 28 days.
18. 16 yr., 3 mo., 29 days.
19. 21 yr., 1 mo., 9 da.
20. 64 yr., 9 mo., 1 da.

Page 171.

2. 31 yd., 2 ft., 8 in.
3. 30 bu., 2 pk., 4 qt.
4. 55 gal.
5. 14 hr., 3 min., 30 seconds.
6. 17 cwt., 11 lb., 14 ounces.
7. 49 lb., 9 oz., 12 pwt.
8. 16 rd., 1 yd., 1 ft.
9. 30 lb., 7 oz., 6 dr., 1 sc.
10. 148 cu. yd.
11. 69 sq. yd., 24 sq. in.
12. 31 rea's, 14 quir's, 12 sheets.
13. 54 yd., 2 ft., 8 in.
14. 41 gal., 3 qt., 1 gi.
15. 67 lb., 8 oz., 8 pwt.
16. 45 rd., 4 yd., 2 ft., 6 in.
17. 26 da., 18 hr., 1 min., 20 sec.
18. 87 sq. yd., 8 sq. ft., 6 sq. in.

Page 172.

2. 2 yd., 5½ in.
3. 3 bu., 1 pk., 7½ qt.
4. 3 gal., 3 qt., 1 pt., 3 gi.
5. 4 hr., 28 min., 7 seconds.

6. 2 cwt., 90 lb., 6½ ounces.
7. 2 lb., 8 oz., 6 pwt.
8. 2 lb., 10 oz., 3 dr., 2½ sc.
9. 2 cu. yd., 21½ cu. ft.
10. 7 sq. yd., 3 sq. ft., 22 sq. in.

Page 174.

2. 270 sq. ft.
3. 400 sq. ft.
4. 42 sq. yd.
5. 32 sq. yd.
6. \$5.40.
7. 47 sq. yd.
8. \$27.60.
9. 33½ A.
10. 50 A.
11. \$15.
12. 80 rods.

Page 175.

2. 72 cu. ft.

Page 176.

3. 160 cu. ft.
4. 175 cu. ft.
5. 216 cu. ft.
6. 640 cu. ft.
7. 12 cords.
8. 5½ cords.
9. 80 ft.
10. 136½ cu. yd.
11. \$74.66½.
12. 8½ perch.

Page 177.

1. 13½ feet.
2. 24 feet.
3. 12½ feet.
4. 66½ feet.
5. 280 feet.
6. \$10.24.
7. \$13.20.

Page 179.

1. 7 yd., 2 ft.
2. \$3.37½.

3. \$4.05.
4. \$75.60.
5. 2607 sheets.
6. 240 pills.
7. \$10.50.
8. 49 bu., 2 pk., 3 qt.
9. 525600 min.
10. $3406\frac{1}{4}$.
11. 2205 pounds.
12. \$57.60.
13. \$222.22 $\frac{1}{2}$.
14. \$22500.
15. 152 gal., 1 pt.

Page 180.

16. 8 da., 2 hr., 6 min., 40 sec.
17. 394 gal., 2 qt.
18. \$138.60.
19. 315 feet.
20. \$35.20.
21. \$17.10.
22. 1 lb., 11 oz., 1 pwt., 18 gr.
23. \$7.05.
24. 72 cu. ft.
25. \$1.50.
26. 71 A., $151\frac{1}{2}$ sq. rd.
27. $26\frac{1}{2}$ yd.
28. November 3, 1783.
29. 1240 gr. iron.
30. 30 balls.

Page 181.

31. \$12.
32. \$79.375.
33. 2 da., 7 hr., 46 min., 40 sec.
34. $47\frac{1}{4}$ pieces.
35. 12 rd., 13 ft., $2\frac{1}{2}$ in.
36. $26\frac{1}{2}$ sq. yd.
37. 1808 miles.

Page 184.

2. \$1.869.
3. \$2.46.
4. \$7.126.
5. 106 gal.
6. 112 mi.

7. 162 inches.
8. 110 sheep.
9. \$8400.

Page 185.

10. \$15600.
11. \$2030.
12. \$1660.

Page 186.

2. 25%.
3. 20%.
4. $33\frac{1}{3}$ %.
5. 75%.
6. $62\frac{1}{2}$ %.
7. $52\frac{1}{2}$ %.
8. 25%.
9. $16\frac{2}{3}$ %.
10. $38\frac{1}{3}$ %.
11. $33\frac{1}{3}$ %.
12. 20%.
13. 75%.
14. 20%.
15. 75%.
16. 80%.

Page 187.

2. 1248.
3. 625.
4. 1260.
5. 3250.
6. \$345.84.
7. 141 bushels.
8. 1500 men.
9. 968 oxen.
10. \$1000.
11. 960 bushels.
12. \$7200.
13. \$400.
14. \$7200.
15. \$15600.

Page 190.

2. \$2.894 +.
3. \$3.823 +.
4. \$1.924.
5. \$1.62 +.
6. \$8.73.
7. \$2.533 +.

8. \$5.36 +.
9. \$6.042 +.
10. \$3.751 +.
11. \$10.707 +.
12. \$1.631 +.
13. \$965 +.
14. \$1.823 +.
15. \$1.168 +.
16. \$.891 +.
17. \$.659 +.
18. \$10.316 +.
19. \$11.131.
20. \$12.293 +.
21. \$17.165 +.
22. \$27.54 +.
23. \$39.715 +.
24. \$69.05 +.
25. \$70.036 +.
26. \$38.536 +.
27. \$1148.
28. \$81.52 +.

Page 191.

1. \$50.312 +.
2. \$1774.
4. 120.8 bushels.
5. \$2.75.
6. \$21.577 +.
7. 48 sq. yd.
8. $13\frac{1}{2}$ acres.
9. \$1.05.
10. 49 years.
11. 98 feet.

Page 192.

12. 20 cents, one;
- 24 cents, other.
13. 10 apples, one;
- 17 apples, other.
14. 6 boys.
15. $2\frac{1}{2}$ days.
16. $2\frac{1}{4}$ days.
17. \$125.
18. \$280.
19. \$58.42 +.
20. \$378.399 +.
21. \$359.101 +.
22. 102 gal., 1 pt.
23. 17 gal., 3 qt., 1 pt.
24. 2129 sheets.

Page 193.

- 25. 12 sheep.
- 26. \$172.50.
- 27. \$200.
- 28. \$423.
- 29. \$62.40.
- 30. 285 bushels.
- 31. \$1916.25.
- 32. \$3108
- 33. $1\frac{1}{175}$.

34. \$5000.

35. $5\frac{1}{4}$.36. $6\frac{1}{4}$.

37. \$60.

38. \$1.00.

39. \$157.72.

Page 194.

40. 9 days.

41. \$131.666 +.

42. $42\frac{1}{2}$ weeks.

43. \$70000.

44. 80 and 60.

45. $37\frac{1}{2}$ days.

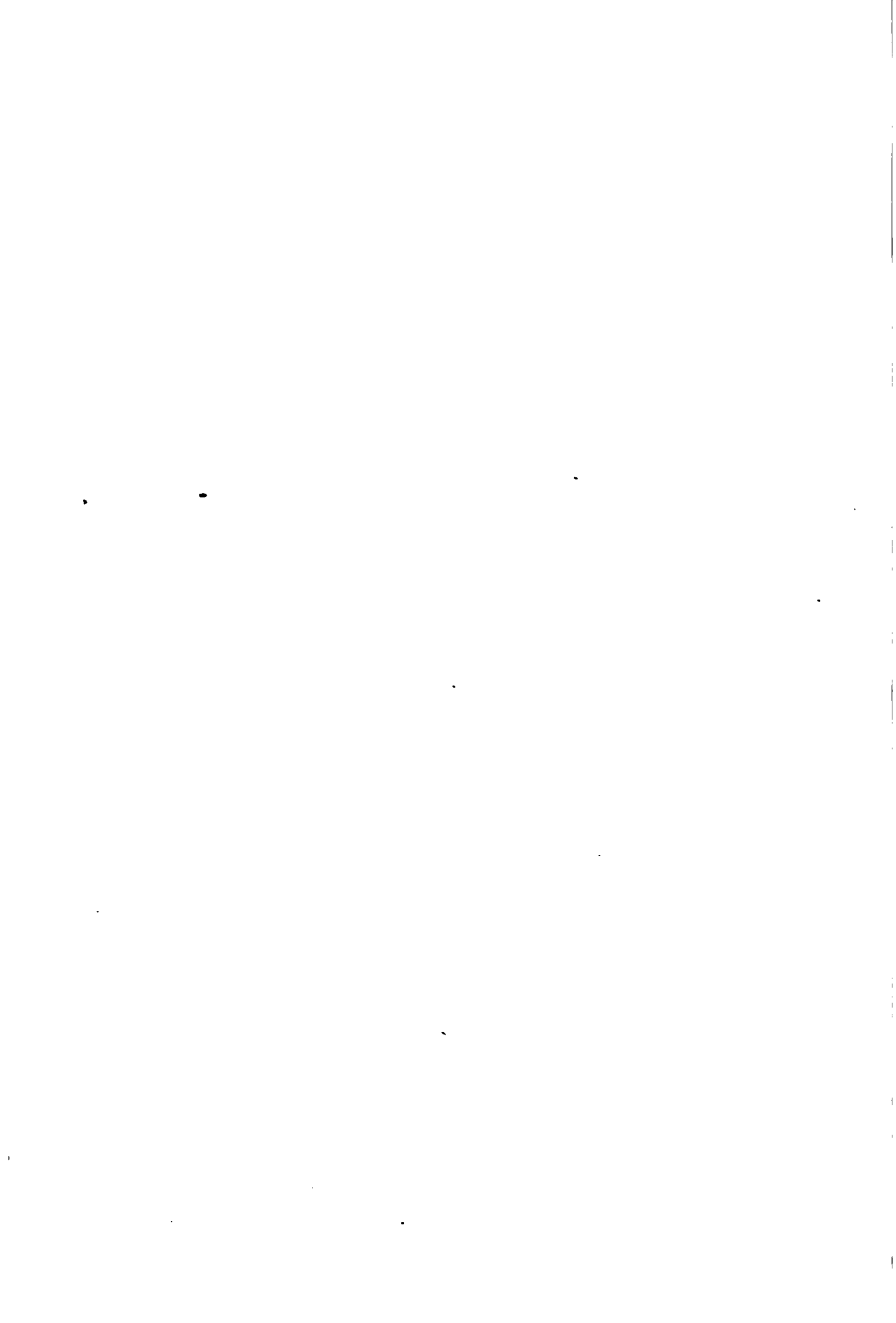
46. \$3200.

47. \$62500.

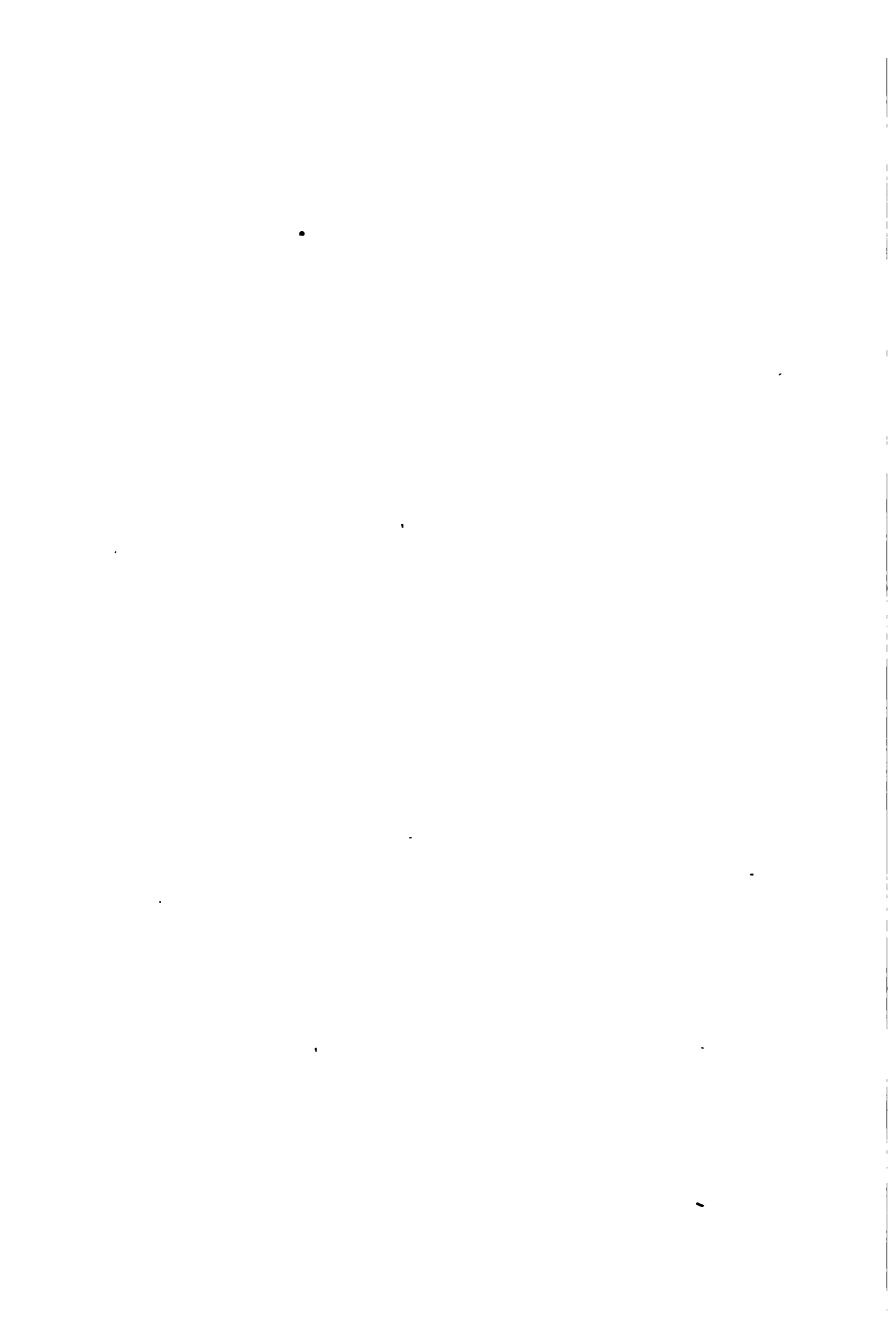
48. \$1.55.

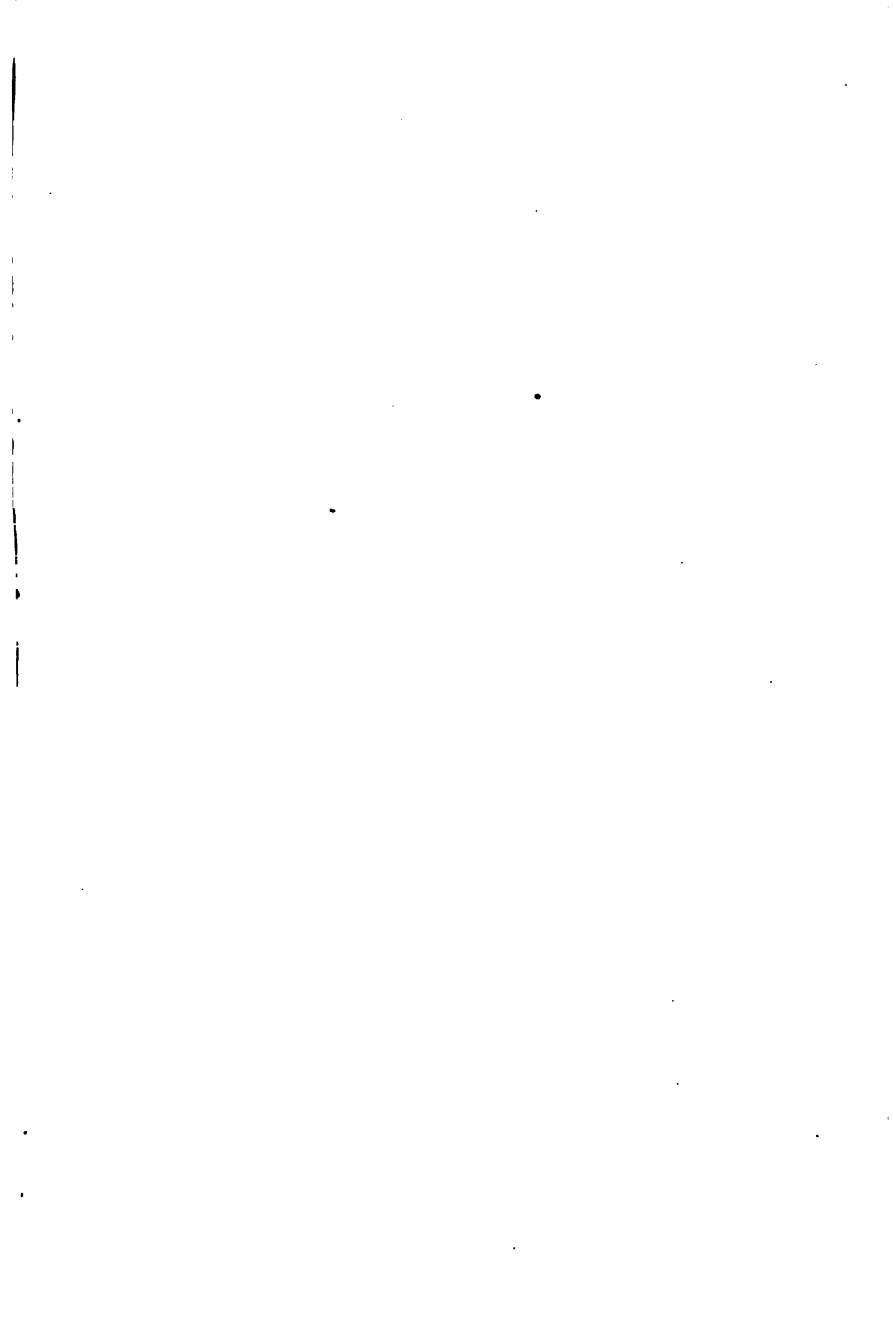
49. \$344.

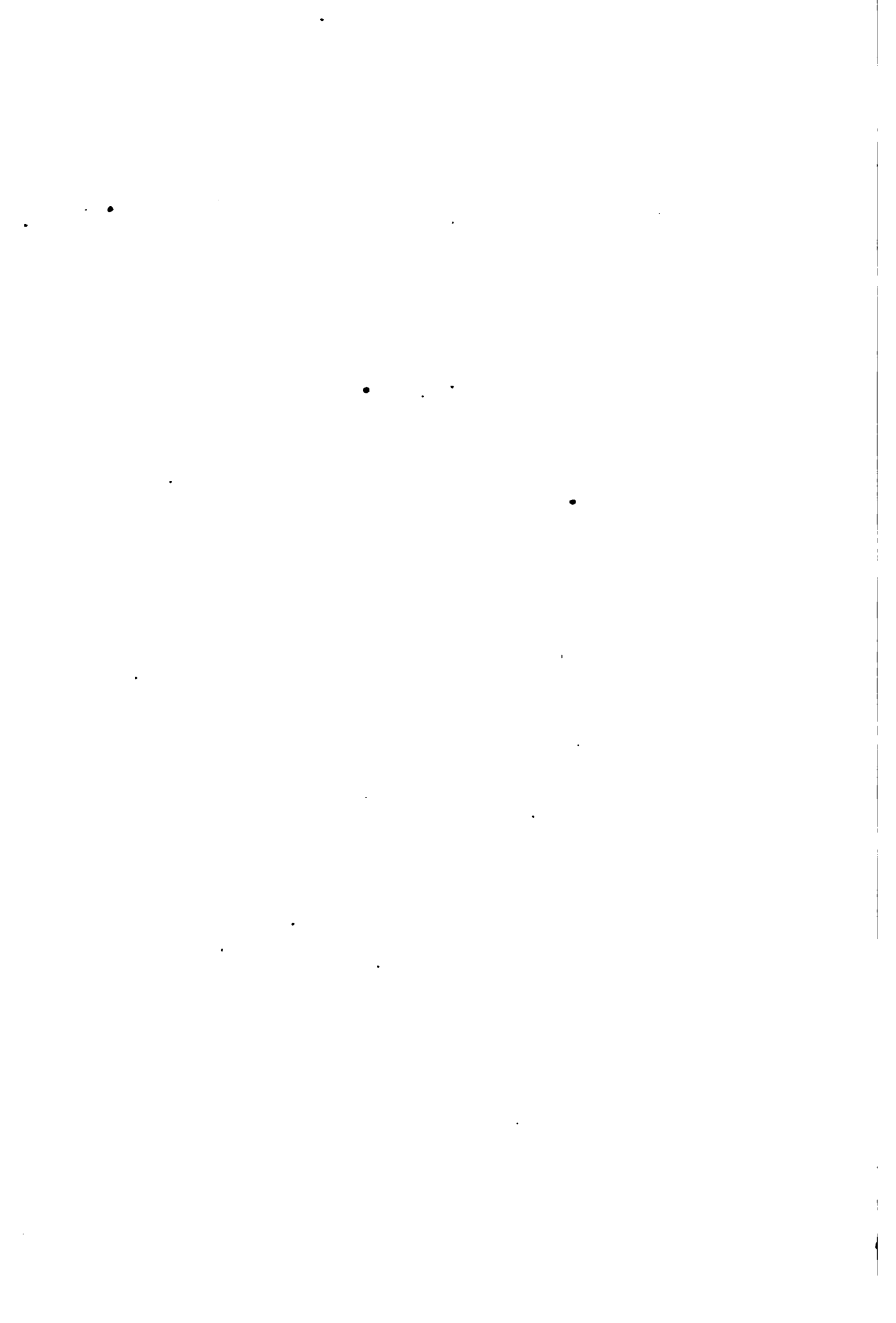
50. 100000 trees.

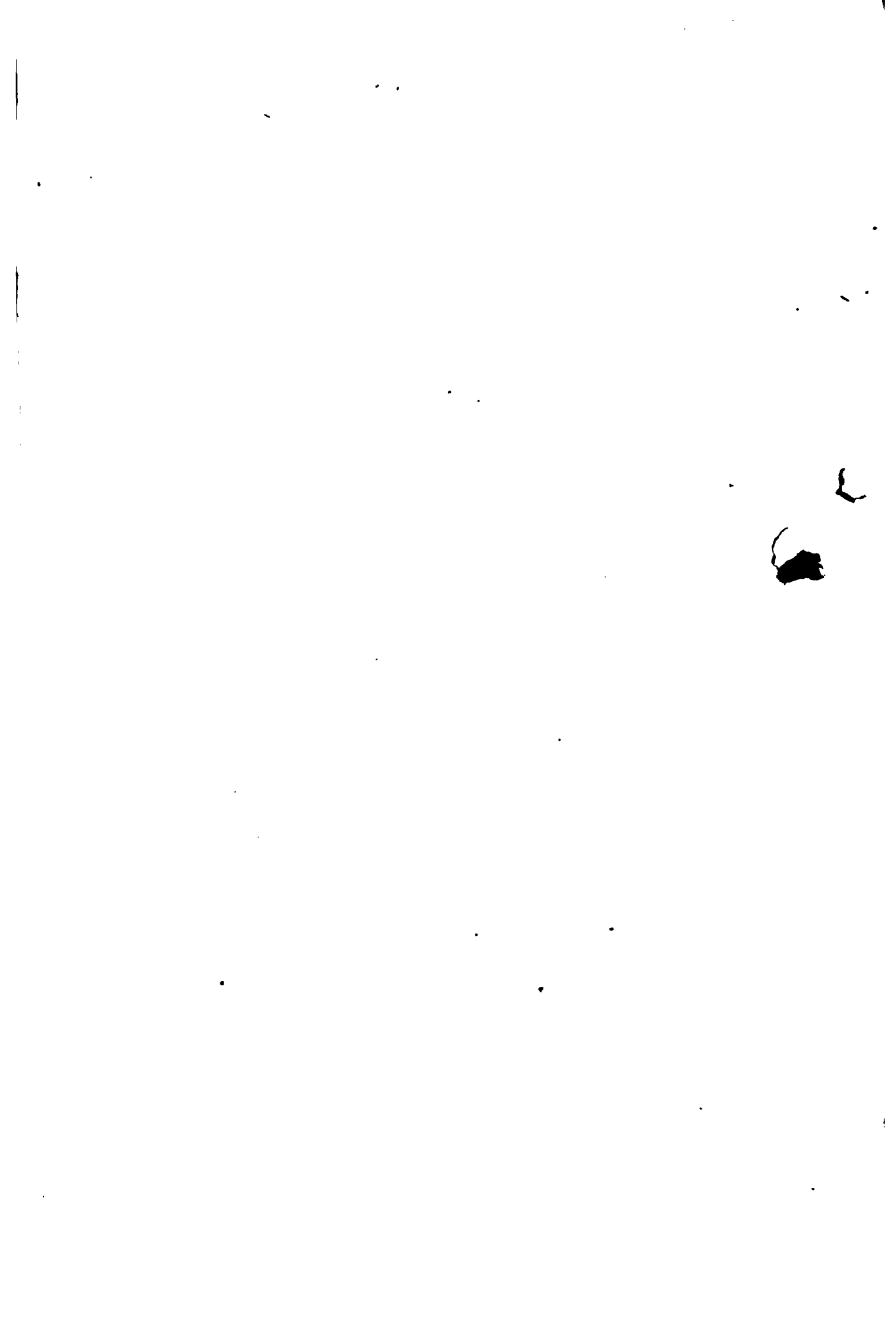












To avoid fine, this book should be returned on
or before the date last stamped below

SON-9.40

APR 01 2002

mi

Milne, William J.
Elementary arithmetic.

LIBRARY. SCHOOL OF EDUCATION, STANFORD

615055

ON
VERSIT

TX
511
156

MILNE'S
ELEMENTARY ARITHMETIC

